1 Advertising on Search Engines

2 Basic Models

3 Advanced Models
Commonly on Search Engines.

Purpose: focused advertisement.
- Better than SPAM (sent to everyone).

Idea: people searching for a keyword may be interested in related products.
Advertising on Baidu

Outline
Advertising on Search Engines
Basic Models
Advanced Models

Advertising on Baidu

Sponsored Search Auction
For each keyword,

- $K$ slots to sell for placing ads
- Each slot can generate a certain number of clicks
Sponsored Search Auction

Three components:
- Advertisers: submit bids for keywords
- Search engines: pose advertisements alongside with search results
- Users: search for keywords; click ad links

Pay-per-click
Basic Setting

\( n \) player, \( v_i \), \( k \) slots, CTR \( c_j \), \( n > k \), one keyword

Assumptions:

1. Player rational
2. No budget
3. Exact match
4. Full information
Truthful Mechanism

- The allocation/payment protocol should make it optimal for each agent to bid their true values.
- In a truthful auction, knowledge about other bids is irrelevant.
VCG Mechanism

- Truthful mechanism
- Generalize Vickrey Auction for multiple different item auction
- Protocol: each player in the auction pays the opportunity cost that their presence introduces to all the other players.

Example:
- Auction a pen and a pencil
- Three bidders A, B, C
- A bids 5 for a pen; B bids 2 for a pencil; C bids 6 for both
- Outcome:
  - A wins a pen and pays 4;
  - B wins a pencil and pays 1.
The ads are sorted by bids and priced the next highest bid.

- Highest bidder gets the best slot, paying the second highest bidding price.
- $i$th highest bidder gets the $i$th best one, paying the $i + 1$st highest bidding price

Simple model produces billions of dollars.
GSP is not truthful

- Two slots with 20/10 clicks
- Three bidders with true values 5/4/1
- Value5 gets 20 clicks
  - Pay 4 each
  - Utility = (5-4)*20 = 20
- Value4 gets 10 clicks
  - Pay 1 each
  - Utility = (4-1)*10 = 30
RED will bid lower

- **Value5 gets 10 clicks**
  - Pay 1 each
  - Utility = (5-1)*10 = 40

- **Value4 gets 20 clicks**
  - Pay 2 each
  - Utility = (4-2)*20 = 40

- **Value5 gets better by changing its bid**
The Nash Equilibrium in GSP

- Varian (2006)
  - Introduced the concept of Symmetric Nash Equilibrium (SNE)
- Edelman, Ostrovsky and Schwarz (2006)
  - Introduced the concept of locally envy free equilibrium (LEFE).

Main results: GSP may get more revenue than VCG.
However...

- Advertisers are selfish.
- Will SNE be achieved in reality?
Convergence

The value of per-click is 4.

\[(4 - 2) \times 20 = 40\]

The value of per-click is 5.

\[(5 - 1) \times 10 = 40\]

The value of per-click is 1.

\[1\]
The 2nd slot winner try to manipulate

The first slot winner moves down to optimize its utility

The value of per-click is 4:

\[(4 - 2.5) \times 20 = 29.8\]

The value of per-click is 5:

\[(5 - 2.5) \times 20 = 50\]

The value of per-click is 1:

\[(4 - 1) \times 10 = 30\]
Strategy

- Cary et al. (07'): balanced bidding strategy
- Bu et al. (07'): forward looking strategy
Forward Looking Nash Equilibrium

- Forward looking strategy

\[ F^i(b^{-i}) = \begin{cases} 
  v^i - \frac{c_k}{c_{k-1}}(v^i - b_{k+1}) & 2 \leq k \leq K \\
  v^i & k = 1 \text{ or } k > K 
\end{cases} \]

- Forward looking Nash equilibrium

\[ \begin{cases} 
  b^i = v^i & \text{for } i = 1 \text{ and } i > K, \\
  b^i = \frac{1}{\theta_{i-1}} \left[ \sum_{j=i}^{K} (\theta_{j-1} - \theta_j)v^j + \theta_Kv^{K+1} \right] & \text{for } 2 \leq i \leq K. 
\end{cases} \]

- Property
  - Any bidder’s payment equals to VCG payment.
  - The auctioneer’s revenue is equivalent to using VCG mechanism.
Sponsored search auction converges to a forward looking equilibrium with probability one under randomized adjustment scheme.

**Lowest-First Adjustment Protocol**

<table>
<thead>
<tr>
<th>Algorithm 1 Lowest-First$(K, j, b_1, b_2, \ldots, b_N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. if $(j = 0)$ then</td>
</tr>
<tr>
<td>2. exit</td>
</tr>
<tr>
<td>3. end if</td>
</tr>
<tr>
<td>4. Let $i$ be the ID of the bidder whose current bid</td>
</tr>
<tr>
<td>is $b_j$ (and equivalently, $b^i$).</td>
</tr>
<tr>
<td>5. Let $h = O^i(M^i(b^{-i}), b^{-i})$.</td>
</tr>
<tr>
<td>6. Let $F^i(b^{-i})$ be the best response function</td>
</tr>
<tr>
<td>value for Bidder $i$.</td>
</tr>
<tr>
<td>7. Re-sort the bid sequence. (So $h$ is the slot of</td>
</tr>
<tr>
<td>the new bid $F^i(b^{-i})$ of Bidder $i$.)</td>
</tr>
<tr>
<td>8. if $(h &lt; j)$ then</td>
</tr>
<tr>
<td>9. call Lowest-First$(K, j, b_1, b_2, \ldots, b_N)$,</td>
</tr>
<tr>
<td>10. else</td>
</tr>
<tr>
<td>11. call Lowest-First$(K, h - 1, b_1, b_2, \ldots, b_N)$</td>
</tr>
<tr>
<td>12. end if</td>
</tr>
</tbody>
</table>
Robustness: Cooperative or Vindicative?

- Cooperative bidder: maximize its own utility only.
- Vindicative bidder: minimize its competitor’s utility.

Liang and Q (07’), Zhou and Lukose (07’), Vorobeyehik and Reeves (07’)

Qi Qi
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Cross Markets Arbitrage: Traffic Arbitrage (Bu et al. (08'))

Energizing Rose Mask - At-Home Facial Treatment Masks - iVillage ...
With just a cup of plain yogurt and a few choice ingredients you can change the look and feel of your skin.
beauty.ivillage.com/skinbody/facecare/0,96sswrmc,00.html - 54k - Cached - Similar pages - Note this

Indian Rose Mask - Craftbits.com Craft Project
Indian Rose Mask craft project, Free Craft Projects, Ideas, Crafts, Kids, Kids lesson plans, Wedding Crafts, Candle and Soap Making, Scrapbook, Crochet, ...
www.craftbits.com/viewProject.do?projectID=764 - 71k - Cached - Similar pages - Note this

Sephora: Korres Wild Rose Mask: Masks
What it is: An instant brightening and illuminating vitamin C mask. What it is formulated to do: Suitable for all skin types, an instant brightening effect ...
www.sephora.com/browse/product.jhtml?id=P155112&categoryid=C11245 - 63k - Cached - Similar pages - Note this

Sponsored Links
Costume Rose
Costume rose
Shop Target.com
www.Target.com

Rose Mask
Bargain Prices. Smart Deals.
Save on Rose Mask.
BizRate.com

Rose Mask For Less
Looking for Rose Mask?
Buy direct from sellers and save.
www.eBay.com

Mask Rose
Huge selection of Mask Rose items.
Yahoo.com
Cross Markets Arbitrage: Click Arbitrage

- The search engine A (or an affiliate) furtively bids a slot from another search engine B’s auction for participant $i$ of A. If a user clicks on the advertisement on B, he/she will be directed to A then redirected to the clients webpage.
- If A pays less to B than it collects from $i$, the act would gain it extra revenue.
There doesn't exist any mechanism which is truthful, multiple-bidding proof, social efficiency and individual rationality.
Leme and Tardos (10’) Lucier and Leme (11’)
Revenue Maximization

Lucier et al. (12')
Topics

1. Not full information: unknown CTR, or dependent CTR
2. Not exact match
3. Limited budget
4. Expressive bidding language