Solutions to Further Problems

Risk Management and Financial Institutions

Third Edition

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Preface

This manual contains answers to all the Further Questions at the ends of the chapters. A separate pdf file contains notes on the teaching of the chapters that some instructors might find useful. Several hundred PowerPoint slides can be downloaded from my website www.rotman.utoronto.ca/~hull or from the Wiley Instructor Resource Center. A sample course outline is also available from these two sources. Any comments or suggestions on the book or this manual or my slides would be appreciated. My e-mail address is hull@rotman.utoronto.ca
Chapter 1: Introduction

1.15. Suppose that one investment has a mean return of 8% and a standard deviation of return of 14%. Another investment has a mean return of 12% and a standard deviation of return of 20%. The correlation between the returns is 0.3. Produce a chart similar to Figure 1.2 showing alternative risk-return combinations from the two investments.

The impact of investing $w_1$ in the first investment and $w_2 = 1 - w_1$ in the second investment is shown in the table below. The range of possible risk-return trade-offs is shown in figure below.

<table>
<thead>
<tr>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$\mu_P$</th>
<th>$\sigma_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>12%</td>
<td>20%</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8</td>
<td>11.2%</td>
<td>17.05%</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>10.4%</td>
<td>14.69%</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
<td>9.6%</td>
<td>13.22%</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>8.8%</td>
<td>12.97%</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0</td>
<td>8.0%</td>
<td>14.00%</td>
</tr>
</tbody>
</table>

1.16. The expected return on the market is 12% and the risk-free rate is 7%. The standard deviation of the return on the market is 15%. One investor creates a portfolio on the efficient frontier with an expected return of 10%. Another creates a portfolio on the efficient frontier with an expected return of 20%. What is the standard deviation of the returns of the two portfolios?
In this case the efficient frontier is as shown in the figure below. The standard deviation of returns corresponding to an expected return of 10% is 9%. The standard deviation of returns corresponding to an expected return of 20% is 39%.

1.17.  
*A bank estimates that its profit next year is normally distributed with a mean of 0.8% of assets and the standard deviation of 2% of assets. How much equity (as a percentage of assets) does the company need to be (a) 99% sure that it will have a positive equity at the end of the year and (b) 99.9% sure that it will have positive equity at the end of the year? Ignore taxes.

(a) The bank can be 99% certain that profit will better than 0.8 − 2.33 × 2 or −3.85% of assets. It therefore needs equity equal to 3.85% of assets to be 99% certain that it will have a positive equity at the year end.

(b) The bank can be 99.9% certain that profit will be greater than 0.8 − 3.09 × 2 or −5.38% of assets. It therefore needs equity equal to 5.38% of assets to be 99.9% certain that it will have a positive equity at the year end.

1.18.  
*A portfolio manager has maintained an actively managed portfolio with a beta of 0.2. During the last year, the risk-free rate was 5% and major equity indices performed very badly, providing returns of about −30%. The portfolio manager produced a return of −10% and claims that in the circumstances it was good. Discuss this claim.

When the expected return on the market is −30% the expected return on a portfolio with a beta of 0.2 is

\[ 0.05 + 0.2 \times (-0.30 - 0.05) = -0.02 \]

or −2%. The actual return of −10% is worse than the expected return. The portfolio manager has achieved an alpha of −8%!
Chapter 2: Banks

2.15. 
Regulators calculate that DLC bank (see Section 2.2) will report a profit that is normally distributed with a mean of $0.6 million and a standard deviation of $2.0 million. How much equity capital in addition to that in Table 2.2 should regulators require for there to be a 99.9% chance of the capital not being wiped out by losses?

There is a 99.9% chance that the profit will not be worse than $0.6 - 3.090 \times 2.0 = -$5.58 million. Regulators will require $0.58 million of additional capital.

2.16. 
*Explain the moral hazard problems with deposit insurance. How can they be overcome?*

Deposit insurance makes depositors less concerned about the financial health of a bank. As a result, banks may be able to take more risk without being in danger of losing deposits. This is an example of moral hazard. (The existence of the insurance changes the behavior of the parties involved with the result that the expected payout on the insurance contract is higher.) Regulatory requirements that banks keep sufficient capital for the risks they are taking reduce their incentive to take risks. One approach (used in the U.S.) to avoiding the moral hazard problem is to make the premiums that banks have to pay for deposit insurance dependent on an assessment of the risks they are taking.

2.17. 
*The bidders in a Dutch auction are as follows:*

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Number of shares</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>60,000</td>
<td>$50.00</td>
</tr>
<tr>
<td>B</td>
<td>20,000</td>
<td>$80.00</td>
</tr>
<tr>
<td>C</td>
<td>30,000</td>
<td>$55.00</td>
</tr>
<tr>
<td>D</td>
<td>40,000</td>
<td>$38.00</td>
</tr>
<tr>
<td>E</td>
<td>40,000</td>
<td>$42.00</td>
</tr>
<tr>
<td>F</td>
<td>40,000</td>
<td>$42.00</td>
</tr>
<tr>
<td>G</td>
<td>50,000</td>
<td>$35.00</td>
</tr>
<tr>
<td>H</td>
<td>50,000</td>
<td>$60.00</td>
</tr>
</tbody>
</table>

The number of shares being auctioned is 210,000. What is the price paid by investors? How many shares does each investor receive?

When ranked from highest to lowest the bidders are B, H, C, A, E and F, D, and G. Individuals B, H, C, and A bid for 160,000 shares in total. Individuals E and F bid for a further 80,000 shares. The price paid by the investors is therefore the price bid by E and F (i.e., $42). Individuals B, H, C, and A get the whole amount of the shares they bid for. Individuals E and F gets 25,000 shares each.
2.18.

An investment bank has been asked to underwrite an issue of 10 million shares by a company. It is trying to decide between a firm commitment where it buys the shares for $10 per share and a best efforts where it charges a fee of 20 cents for each share sold. Explain the pros and cons of the two alternatives.

If it succeeds in selling all 10 million shares in a best efforts arrangement, its fee will be $2 million. If it is able to sell the shares for $10.20, this will also be its profit in a firm commitment arrangement. The decision is likely to hinge on a) an estimate of the probability of selling the shares for more than $10.20 and b) the investment banks appetite for risk. For example, if the bank is 95% certain that it will be able to sell the shares for more than $10.20, it is likely to choose a firm commitment. But if assesses the probability of this to be only 50% or 60% it is likely to choose a best efforts arrangement.
Chapter 3: Insurance Companies and Pension Funds

3.16. (Spreadsheet Provided).
Use Table 3.1 to calculate the minimum premium an insurance company should charge for a $5 million three-year term life insurance contract issued to a man aged 60. Assume that the premium is paid at the beginning of each year and death always takes place halfway through a year. The risk-free interest rate is 6% per annum (with semiannual compounding).

The unconditional probability of the man dying in years one, two, and three can be calculated from Table 3.1 as follows:

Year 1: 0.011407

Year 2: (1−0.011407) × 0.012315 = 0.012175

Year 3: (1−0.011407)(1−0.01217) × 0.013289 = 0.012976

The expected payouts at times 0.5, 1.5, 2.5 are therefore $57,035.00, $60,872.61, and $64,878.13. These have a present value of $167,045.29. The survival probability of the man is

Year 0: 1

Year 1: 1−0.011407 = 0.988593

Year 2: 1−0.011407−0.01217 = 0.976418

The present value of the premiums received per dollar of premium paid per year is therefore 2.799379. The minimum premium is

\[
\frac{167,045.29}{2.799379} = 59,672.27
\]

or $59,672.27.

3.17
An insurance company’s losses of a particular type per year are to a reasonable approximation normally distributed with a mean of $150 million and a standard deviation of $50 million. (Assume that the risks taken on by the insurance company are entirely non-systematic.) The one-year risk-free rate is 5% per annum with annual compounding. Estimate the cost of the following:

(a) A contract that will pay in one-year’s time 60% of the insurance company’s costs on a pro rata basis

(b) A contract that pays $100 million in one-year’s time if losses exceed $200 million.

(a) The losses in millions of dollars are normally distributed with mean 150 and standard deviation 50. The payout from the reinsurance contract is therefore normally distributed with
mean 90 and standard deviation 30. Assuming that the reinsurance company feels it can diversify away the risk, the minimum cost of reinsurance is

\[
\frac{90}{1.05} = 85.71
\]

or $85.71 million.

(b) The probability that losses will be greater than $200 million is the probability that a normally distributed variable is greater than one standard deviation above the mean. This is 0.1587. The expected payoff in millions of dollars is therefore \(0.1587 \times 100 = 15.87\) and the value of the contract is

\[
\frac{15.87}{1.05} = 15.11
\]

or $15.11 million.

3.18. 
During a certain year, interest rates fall by 200 basis points (2%) and equity prices are flat. Discuss the effect of this on a defined benefit pension plan that is 60% invested in equities and 40% invested in bonds.

The value of a bond increases when interest rates fall. The value of the bond portfolio should therefore increase. However, a lower discount rate will be used in determining the value of the pension fund liabilities. This will increase the value of the liabilities. The net effect on the pension plan is likely to be negative. This is because the interest rate decrease affects 100% of the liabilities and only 40% of the assets.

3.19. (Spreadsheet Provided)
Suppose that in a certain defined benefit pension plan

(a) Employees work for 45 years earning wages that increase at a real rate of 2%
(b) They retire with a pension equal to 70% of their final salary. This pension increases at the rate of inflation minus 1%
(c) The pension is received for 18 years.
(d) The pension fund’s income is invested in bonds which earn the inflation rate plus 1.5%.

Estimate the percentage of an employee’s salary that must be contributed to the pension plan if it is to remain solvent. (Hint: Do all calculations in real rather than nominal dollars.)
The salary of the employee makes no difference to the answer. (This is because it has the effect of scaling all numbers up or down.) If we assume the initial salary is $100,000 and that the real growth rate of 2% is annually compounded, the final salary at the end of 45 years is $239,005.31. The spreadsheet is used in conjunction with Solver to show that the required contribution rate is 25.02% (employee plus employer). The value of the contribution grows to $2,420,354.51 by the end of the 45 year working life. (This assumes that the real return of 1.5% is annually compounded.) This value reduces to zero over the following 18 years under the assumptions made. This calculation confirms the point made in Section 3.12 that defined benefit plans require higher contribution rates that those that exist in practice.
Chapter 4: Mutual Funds and Hedge Funds

4.15. An investor buys 100 shares in a mutual fund on January 1, 2012, for $50 each. The fund earns dividends of $2 and $3 per share during 2012 and 2013. These are reinvested in the fund. Its realized capital gains in 2012 and 2013 are $5 per share and $3 per share, respectively. The investor sells the shares in the fund during 2014 for $59 per share. Explain how the investor is taxed.

The investor pays tax on dividends of $200 and $300 in year 2012 and 2013, respectively. The investor also has to pay tax on realized capital gains by the fund. This means tax will be paid on capital gains of $500 and $300 in year 2012 and 2013, respectively. The result of all this is that the basis for the shares increases from $50 to $63. The sale at $59 in year 2014 leads to a capital loss of $4 per share or $400 in total.

4.16. Good years are followed by equally bad years for a mutual fund. It earns +8%, –8%, +12%, –12% in successive years. What is the investor’s overall return for the four years?

The investors overall return is

\[1.08 \times 0.92 \times 1.12 \times 0.88 - 1 = -0.0207\]

or –2.07% for the four years.

4.17. A fund of funds divides its money between five hedge funds that earn –5%, 1%, 10%, 15%, and 20% before fees in a particular year. The fund of funds charges 1 plus 10% and the hedge funds charge 2 plus 20%. The hedge funds’ incentive fees are calculated on the return after management fees. The fund of funds incentive fee is calculated on the net (after management fees and incentive fees) average return of the hedge funds in which it invests and after its own management fee has been subtracted. What is the overall return on the investments? How is it divided between the fund of funds, the hedge funds, and investors in the fund of funds?

The overall return on the investments is the average of –5%, 1%, 10%, 15%, and 20% or 8.2%. The hedge fund fees are 2%, 2%, 3.6%, 4.6%, and 5.6%. These average 3.56%. The returns earned by the fund of funds after hedge fund fees are therefore –7%, –1%, 6.4%, 10.4%, and 14.4%. These average 4.64%. The fund of funds fee is 1% + 0.364% or 1.364% leaving 3.276% for the investor. The return earned is therefore divided as shown in the table below.
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return earned by hedge funds</td>
<td>8.200%</td>
</tr>
<tr>
<td>Fees to hedge funds</td>
<td>3.560%</td>
</tr>
<tr>
<td>Fees to fund of funds</td>
<td>1.364%</td>
</tr>
<tr>
<td>Return to investor</td>
<td>3.276%</td>
</tr>
</tbody>
</table>

4.18. 
A hedge funds charges 2 plus 20%. A pension fund invests in the hedge fund. Plot the return to the pension fund as a function of the return to the hedge fund.

The plot is shown in the chart below. If the hedge fund return is less than 2%, the pension fund return is 2% less than the hedge fund return. If it is greater than 2%, the pension fund return is less than the hedge fund return by 2% plus 20% of the excess of the return above 2%.
Chapter 5: Trading in Financial Markets

5.30.
A company enters into a short futures contract to sell 5,000 bushels of wheat for 250 cents per bushel. The initial margin is $3,000 and the maintenance margin is $2,000. What price change would lead to a margin call? Under what circumstances could $1,500 be withdrawn from the margin account?

There is a margin call when more than $1,000 is lost from the margin account. This happens when the futures price of wheat rises by more than 1,000/5,000 = 0.20. There is a margin call when the futures price of wheat rises above 270 cents. An amount, $1,500, can be withdrawn from the margin account when the futures price of wheat falls by 1,500/5,000 = 0.30. The withdrawal can take place when the futures price falls to 220 cents.

5.31.
The current price of a stock is $94, and three-month European call options with a strike price of $95 currently sell for $4.70. An investor who feels that the price of the stock will increase is trying to decide between buying 100 shares and buying 2,000 call options (= 20 contracts). Both strategies involve an investment of $9,400. What advice would you give? How high does the stock price have to rise for the option strategy to be more profitable?

The investment in call options entails higher risks but can lead to higher returns. If the stock price stays at $94, an investor who buys call options loses $9,400 whereas an investor who buys shares neither gains nor loses anything. If the stock price rises to $120, the investor who buys call options gains
\[ 2000 \times (120 - 95) - 9400 = 40,600 \]
An investor who buys shares gains
\[ 100 \times (120 - 94) = 2,600 \]
The strategies are equally profitable if the stock price rises to a level, \( S \), where
\[ 100 \times (S - 94) = 2000(S - 95) - 9400 \]
or
\[ S = 100 \]
The option strategy is therefore more profitable if the stock price rises above $100.

5.32.
A bond issued by Standard Oil worked as follows. The holder received no interest. At the bond’s maturity the company promised to pay $1,000 plus an additional amount based on the price of oil at that time. The additional amount was equal to the product of 170 and the excess (if any) of the price of a barrel of oil at maturity over $25. The maximum additional amount paid was $2,550 (which corresponds to a price of $40 per barrel). Show that the bond is a combination of a regular bond, a long position in call options on oil with a strike price of $25, and a short position in call options on oil with a strike price of $40.

Suppose \( S_T \) is the price of oil at the bond’s maturity. In addition to $1000 the Standard Oil bond pays:
\[ S_t < \$25 : 0 \]
\[ S_t > \$25 : 170 (S_t - 25) \]
\[ S_t > \$40 : 2, 550 \]

This is the payoff from 170 call options on oil with a strike price of 25 less the payoff from 170 call options on oil with a strike price of 40. The bond is therefore equivalent to a regular bond plus a long position in 170 call options on oil with a strike price of $25 plus a short position in 170 call options on oil with a strike price of $40. The investor has what is termed a bull spread on oil.

5.33.
The price of gold is currently $1,500 per ounce. The forward price for delivery in one year is $1,700. An arbitrageur can borrow money at 10% per annum. What should the arbitrageur do? Assume that the cost of storing gold is zero and that gold provides no income.

The arbitrageur could borrow money to buy 100 ounces of gold today and short futures contracts on 100 ounces of gold for delivery in one year. This means that gold is purchased for $1,500 per ounce and sold for $1,700 per ounce. The return (about 13% per annum) is greater than the 10% cost of the borrowed funds. This is such a profitable opportunity that the arbitrageur should buy as many ounces of gold as possible and short futures contracts on the same number of ounces. Unfortunately, arbitrage opportunities as profitable as this rarely, if ever, arise in practice.

5.34.
A company’s investments earn LIBOR minus 0.5%. Explain how it can use the quotes in Table 5.5 to convert them to (a) three-, (b) five-, and (c) ten-year fixed-rate investments.

(a) By entering into a three-year swap where it receives 6.21% and pays LIBOR the company earns 5.71% for three years.
(b) By entering into a five-year swap where it receives 6.47% and pays LIBOR the company earns 5.97% for five years.
(c) By entering into a swap where it receives 6.83% and pays LIBOR for ten years the company earns 6.33% for ten years.

5.35.
What position is equivalent to a long forward contract to buy an asset at \( K \) on a certain date and a long position in a European put option to sell it for \( K \) on that date?

The position is the same as a European call to buy the asset for \( K \) on the date.

5.36.
Estimate the interest rate paid by P&G on the 5/30 swap in Business Snapshot 5.4 if (a) the CP rate is 6.5% and the Treasury yield curve is flat at 6% and (b) the CP rate is 7.5% and the Treasury yield curve is flat at 7% with semiannual compounding.
(a) When the CP rate is 6.5% and Treasury rates are 6% with semiannual compounding, the
CMT% is 6% and an Excel spreadsheet can be used to show that the price of a 30-year bond with
a 6.25% coupon is about 103.46. The spread is zero and the rate paid by P&G is 5.75%.
(b) When the CP rate is 7.5% and Treasury rates are 7% with semiannual compounding, the
CMT% is 7% and the price of a 30-year bond with a 6.25% coupon is about 90.65. The spread is
therefore
\[
\max[0, (98.5 \times 7/5.78 - 90.65)/100]
\]
or 28.64%. The rate paid by P&G is 35.39%.

5.37.
A trader buys 200 shares of a stock on margin. The price of the stock is $20. The initial margin is
60% and the maintenance margin is 30%. How much money does the trader have to provide
initially? For what share price is there a margin call?

The trader has to provide 60% of the price of the stock or $2,400. There is a margin call when
the margin account balance as a percent of the value of the shares falls below 30%. When the
share price is \( S \) the margin account balance is \( 2400 + 200 \times (S-20) \) and the value of the position
is \( 200 \times S \). There is a margin call when
\[
2400 + 200 \times (S-20) < 0.3 \times 200 \times S
\]
or
\[
140 \times S < 1600
\]
or
\[
S < 11.43
\]
that is, when the stock price is less than $11.43.
Chapter 6: The Credit Crisis of 2007

6.14. Suppose that the principal assigned to the senior, mezzanine, and equity tranches for the ABSs and ABS CDO in Figure 6.4 is 70%, 20%, and 10% instead of 75%, 20% and 5%. How are the results in Table 6.1 affected?

<table>
<thead>
<tr>
<th>Losses to subprime portfolio</th>
<th>Losses to Mezz tranche of ABS</th>
<th>Losses to equity tranche of ABS CDO</th>
<th>Losses to Mezz tranche of ABS CDO</th>
<th>Losses to senior tranche of ABS CDO</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>15%</td>
<td>25%</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>20%</td>
<td>50%</td>
<td>100%</td>
<td>100%</td>
<td>28.6%</td>
</tr>
<tr>
<td>25%</td>
<td>75%</td>
<td>100%</td>
<td>100%</td>
<td>60%</td>
</tr>
</tbody>
</table>

6.15. Investigate what happens as the width of the mezzanine tranche of the ABS in Figure 6.4 is decreased, with the reduction in the mezzanine tranche principal being divided equally between the equity and senior tranches. In particular, what is the effect on Table 6.1?

The ABS CDO tranches become similar to each other. Consider the situation where the tranche widths are 14%, 2%, and 84% for the equity, mezzanine, and senior tranches. The table becomes:

<table>
<thead>
<tr>
<th>Losses to subprime portfolio</th>
<th>Losses to Mezz tranche of ABS</th>
<th>Losses to equity tranche of ABS CDO</th>
<th>Losses to Mezz tranche of ABS CDO</th>
<th>Losses to senior tranche of ABS CDO</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>14%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>15%</td>
<td>50%</td>
<td>100%</td>
<td>100%</td>
<td>33%</td>
</tr>
<tr>
<td>16%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>20%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>25%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Chapter 7: How Traders Manage Their Risks

7.15. The gamma and vega of a delta-neutral portfolio are 50 per $ per $ and 25 per %, respectively. Estimate what happens to the value of the portfolio when there is a shock to the market causing the underlying asset price to decrease by $3 and its volatility to increase by 4%.

With the notation of the text, the increase in the value of the portfolio is

\[ 0.5 \times \text{gamma} \times (\Delta S)^2 + \text{vega} \times \Delta \sigma \]

This is

\[ 0.5 \times 50 \times 3^2 + 25 \times 4 = 325 \]

The result should be an increase in the value of the portfolio of $325.

7.16. Consider a one-year European call option on a stock when the stock price is $30, the strike price is $30, the risk-free rate is 5%, and the volatility is 25% per annum. Use the DerivaGem software to calculate the price, delta, gamma, vega, theta, and rho of the option. Verify that delta is correct by changing the stock price to $30.1 and recomputing the option price. Verify that gamma is correct by recomputing the delta for the situation where the stock price is $30.1. Carry out similar calculations to verify that vega, theta, and rho are correct.

The price, delta, gamma, vega, theta, and rho of the option are 3.7008, 0.6274, 0.050, 0.1135, −0.00596, and 0.1512. When the stock price increases to 30.1, the option price increases to 3.7638. The change in the option price is 3.7638 − 3.7008 = 0.0630. Delta predicts a change in the option price of 0.6274 × 0.1 = 0.0627 which is very close. When the stock price increases to 30.1, delta increases to 0.6324. The size of the increase in delta is 0.6324 − 0.6274 = 0.005. Gamma predicts an increase of 0.050 × 0.1 = 0.005 which is (to three decimal places) the same. When the volatility increases from 25% to 26%, the option price increases by 0.1136 from 3.7008 to 3.8144. This is consistent with the vega value of 0.1135. When the time to maturity is changed from 1 to 1−1/365 the option price reduces by 0.006 from 3.7008 to 3.6948. This is consistent with a theta of −0.00596. Finally, when the interest rate increases from 5% to 6%, the value of the option increases by 0.1527 from 3.7008 to 3.8535. This is consistent with a rho of 0.1512.

7.17. A financial institution has the following portfolio of over-the-counter options on sterling:

<table>
<thead>
<tr>
<th>Type</th>
<th>Position</th>
<th>Delta of Option</th>
<th>Gamma of Option</th>
<th>Vega of Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call</td>
<td>−1,000</td>
<td>0.50</td>
<td>2.2</td>
<td>1.8</td>
</tr>
<tr>
<td>Call</td>
<td>−500</td>
<td>0.80</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>Put</td>
<td>−2,000</td>
<td>−0.40</td>
<td>1.3</td>
<td>0.7</td>
</tr>
<tr>
<td>Call</td>
<td>−500</td>
<td>0.70</td>
<td>1.8</td>
<td>1.4</td>
</tr>
</tbody>
</table>
A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

(a) What position in the traded option and in sterling would make the portfolio both gamma neutral and delta neutral?
(b) What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral?

The delta of the portfolio is
\[-1,000 \times 0.50 - 500 \times 0.80 - 2,000 \times (-0.40) - 500 \times 0.70 = -450\]
The gamma of the portfolio is
\[-1,000 \times 2.2 - 500 \times 0.6 - 2,000 \times 1.3 - 500 \times 1.8 = -6,000\]
The vega of the portfolio is
\[-1,000 \times 1.8 - 500 \times 0.2 - 2,000 \times 0.7 - 500 \times 1.4 = -4,000\]

(a) A long position in 4,000 traded options will give a gamma-neutral portfolio since the long position has a gamma of 4,000 \(\times 1.5 = +6,000\). The delta of the whole portfolio (including traded options) is then:
\[4,000 \times 0.6 - 450 = 1,950\]
Hence, in addition to the 4,000 traded options, a short position in £1,950 is necessary so that the portfolio is both gamma and delta neutral.

(b) A long position in 5,000 traded options will give a vega-neutral portfolio since the long position has a vega of 5,000 \(\times 0.8 = +4,000\). The delta of the whole portfolio (including traded options) is then
\[5,000 \times 0.6 - 450 = 2,550\]
Hence, in addition to the 5,000 traded options, a short position in £2,550 is necessary so that the portfolio is both vega and delta neutral.

7.18.
Consider again the situation in Problem 7.17. Suppose that a second traded option with a delta of 0.1, a gamma of 0.5, and a vega of 0.6 is available. How could the portfolio be made delta, gamma, and vega neutral?

Let \(w_1\) be the position in the first traded option and \(w_2\) be the position in the second traded option. We require:
\[6,000 = 1.5w_1 + 0.5w_2\]
\[4,000 = 0.8w_1 + 0.6w_2\]
The solution to these equations can easily be seen to be \(w_1 = 3,200, w_2 = 2,400\). The whole portfolio then has a delta of
\[-450 + 3,200 \times 0.6 + 2,400 \times 0.1 = 1,710\]
Therefore the portfolio can be made delta, gamma and vega neutral by taking a long position in 3,200 of the first traded option, a long position in 2,400 of the second traded option and a short position in £1,710.
7.19. (Spreadsheet Provided)
Reproduce Table 7.2. (In Table 7.2, the stock position is rounded to the nearest 100 shares.) Calculate the gamma and theta of the position each week. Using the DerivaGem Applications Builders to calculate the change in the value of the portfolio each week (before the rebalancing at the end of the week) and check whether equation (7.2) is approximately satisfied. (Note: DerivaGem produces a value of theta “per calendar day.” The theta in equation 7.2 is “per year.”)

Consider the first week. The portfolio consists of a short position in 100,000 options and a long position in 52,200 shares. The value of the option changes from $240,053 at the beginning of the week to $188,760 at the end of the week for a gain of $51,293. The value of the shares change from 52,200 × 49 = $2,557,800 to 52,200 × 48.12 = $2,511,864 for a loss of $45,936. The net gain is 51,293 − 45,936 = $5,357. The gamma and theta (per year) of the portfolio are −6,554.4 and 430,533 so that equation (6.2) predicts the gain as

\[ 430,533 \times \frac{1}{52} + 0.5 \times 6,554.4 \times (48.12 - 49)^2 = 5,742 \]

The results for all 20 weeks are shown in the following table.

<table>
<thead>
<tr>
<th>Week</th>
<th>Actual Gain ($)</th>
<th>Predicted Gain ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5,357</td>
<td>5,742</td>
</tr>
<tr>
<td>2</td>
<td>5,689</td>
<td>6,093</td>
</tr>
<tr>
<td>3</td>
<td>−19,742</td>
<td>−21,084</td>
</tr>
<tr>
<td>4</td>
<td>1,941</td>
<td>1,572</td>
</tr>
<tr>
<td>5</td>
<td>3,706</td>
<td>3,652</td>
</tr>
<tr>
<td>6</td>
<td>9,320</td>
<td>9,191</td>
</tr>
<tr>
<td>7</td>
<td>6,249</td>
<td>5,936</td>
</tr>
<tr>
<td>8</td>
<td>9,491</td>
<td>9,259</td>
</tr>
<tr>
<td>9</td>
<td>961</td>
<td>870</td>
</tr>
<tr>
<td>10</td>
<td>−23,380</td>
<td>−18,992</td>
</tr>
<tr>
<td>11</td>
<td>1,643</td>
<td>2,497</td>
</tr>
<tr>
<td>12</td>
<td>2,645</td>
<td>1,356</td>
</tr>
<tr>
<td>13</td>
<td>11,365</td>
<td>10,923</td>
</tr>
<tr>
<td>14</td>
<td>−2,876</td>
<td>−3,342</td>
</tr>
<tr>
<td>15</td>
<td>12,936</td>
<td>12,302</td>
</tr>
<tr>
<td>16</td>
<td>7,566</td>
<td>8,815</td>
</tr>
<tr>
<td>17</td>
<td>−3,880</td>
<td>−2,763</td>
</tr>
<tr>
<td>18</td>
<td>6,764</td>
<td>6,899</td>
</tr>
<tr>
<td>19</td>
<td>4,295</td>
<td>5,205</td>
</tr>
<tr>
<td>20</td>
<td>4,804</td>
<td>4,805</td>
</tr>
</tbody>
</table>
Chapter 8: Interest Rate Risk

8.15. Suppose that a bank has $10 billion of one-year loans and $30 billion of five-year loans. These are financed by $35 billion of one-year deposits and $5 billion of five-year deposits. The bank has equity totaling $2 billion and its return on equity is currently 12%. Estimate what change in interest rates next year would lead to the bank’s return on equity being reduced to zero. Assume that the bank is subject to a tax rate of 30%.

The bank has an asset-liability mismatch of $25 billion. The profit after tax is currently 12% of $2 billion or $0.24 billion. If interest rates rise by $X$% the bank's before-tax loss (in billions of dollars) is $25 \times 0.01 \times X = 0.25X$. After taxes this loss becomes $0.7 \times 0.25X = 0.175X$. The bank’s return on equity would be reduced to zero when $0.175X = 0.24$ or $X = 1.37$. A 1.37% rise in rates would therefore reduce the return on equity to zero.

8.16. Portfolio A consists of a one-year zero-coupon bond with a face value of $2,000 and a 10-year zero-coupon bond with a face value of $6,000. Portfolio B consists of a 5.95-year zero-coupon bond with a face value of $5,000. The current yield on all bonds is 10% per annum (continuously compounded)

(a) Show that both portfolios have the same duration.
(b) Show that the percentage changes in the values of the two portfolios for a 0.1% per annum increase in yields are the same.
(c) What are the percentage changes in the values of the two portfolios for a 5% per annum increase in yields?

(a) The duration of Portfolio A is

$$1 \times 2000e^{-0.1 \times 1} + 10 \times 6000e^{-0.1 \times 10}$$

$$2000e^{-0.1 \times 1} + 6000e^{-0.1 \times 10} = 5.95$$

Since this is also the duration of Portfolio B, the two portfolios do have the same duration.

(b) The value of Portfolio A is

$$2000e^{-0.1 \times 1} + 6000e^{-0.1 \times 10} = 4,016.95$$

When yields increase by 10 basis points its value becomes

$$2000e^{-0.101 \times 1} + 6000e^{-0.101 \times 10} = 3,993.18$$

The percentage decrease in value is

$$\frac{23.77}{4,016.95} \times 100 = 0.59$$

The value of Portfolio B is

$$5000e^{-0.1 \times 5.95} = 2,757.81$$
When yields increase by 10 basis points its value becomes
\[ 5000 \ e^{-0.10 \times 5.95} = 2,741.45 \]
The percentage decrease in value is
\[ \frac{16.36}{2,757.81} \times 100 = 0.59 \%
\]
The percentage changes in the values of the two portfolios for a 10 basis point increase in yields are therefore the same.

(c) When yields increase by 5\% the value of Portfolio A becomes
\[ 2000e^{-0.15 \times 1} + 6000e^{-0.15 \times 10} = 3,060.20 \]
and the value of Portfolio B becomes
\[ 5000e^{0.15 \times 5.95} = 2,048.15 \]
The percentage reductions in the values of the two portfolios are:
Portfolio A:
\[ \frac{956.75}{4,016.95} \times 100 = 23.82 \%
\]
Portfolio B:
\[ \frac{709.66}{2,757.81} \times 100 = 25.73 \%
\]

8.17.
*What are the convexities of the portfolios in Problem 8.16? To what extent does (a) duration and (b) convexity explain the difference between the percentage changes calculated in part (c) of Problem 8.16?*

For Portfolio A the convexity is
\[ 1^2 \times 2000e^{-0.1 \times 1} + 10^2 \times 6000e^{-0.1 \times 10} \]
\[ \frac{2000e^{-0.1 \times 1} + 6000e^{-0.1 \times 10}}{2000e^{-0.1 \times 1} + 6000e^{-0.1 \times 10}} = 55.40 \]
For portfolio B the convexity is 5.95\(^2\) or 35.4025 The percentage change in the two portfolios predicted by the duration measure is the same and equal to \(-5.95 \times 0.05 = -0.2975\) or \(-29.75\%\).
However, the convexity measure predicts that the percentage change in the first portfolio will be \(-5.95 \times 0.05 + 0.5 \times 55.40 \times 0.05^2 = -0.228\)
and that for the second portfolio it will be
\(-5.95 \times 0.05 = 0.5 \times 35.4025 \times 0.05^2 = -0.253\)
Duration does not explain the difference between the percentage changes. Convexity explains part of the difference. 5\% is such a big shift in the yield curve that even the use of the convexity relationship does not give accurate results. Better results would be obtained if a measure involving the third partial derivative with respect to a parallel shift, as well as the first and second, was considered.

8.18.
*When the partial durations are as in Table 8.5, estimate the effect of a shift in the yield curve where the ten-year rate stays the same, the one-year rate moves up by 9e, and the movements in intermediate rates are calculated by interpolation between 9e and 0. How could your answer be calculated from the results for the rotation calculated in Section 8.6?*
The proportional change in the value of the portfolio resulting from the specified shift is
\[-(0.2 \times 9e + 0.6 \times 8e + 0.9 \times 7e + 1.6 \times 6e + 2.0 \times 5e - 2.1 \times 3e) = -26.2e\]
The shift is the same as a parallel shift of 6e and a rotation of −e. (The rotation is of the same magnitude as that considered in the text but in the opposite direction). The total duration of the portfolio is 0.2 and so the percentage change in the portfolio arising from the parallel shift is
\[-0.2 \times 6e = -1.2e\]. The percentage change in the portfolio value arising from the rotation is −25.0e. (This is the same as the number calculated at the end of Section 8.6 but with the opposite sign.) The total percentage change is therefore −26.2e, as calculated from the partial durations.

8.19. (Spreadsheet Provided)

Suppose that the change in a portfolio value for a one-basis-point shift in the 1-year, 2-year, 3-year, 4-year, 5-year, 7-year, 10-year, and 30-year rates are (in $ million) +5, −3, −1, +2, +5, +7, +8, and +1, respectively. Estimate the delta of the portfolio with respect to the first three factors in Table 8.7. Quantify the relative importance of the three factors for this portfolio.

The delta with respect to the first factor is
\[0.215 \times 5 + 0.331 \times (-3) + 0.372 \times (-1) + 0.392 \times 2 + 0.404 \times 5 + 0.394 \times 7 + 0.376 \times 8 + 0.305 \times 1 = 8.590\]
Similarly, the deltas with respect to the second and third factors are 3.804 and 0.472, respectively. The relative importance of the factors can be seen by multiplying the factor exposure by the factor standard deviation. The second factor is about (3.804×4.77)/(8.590×17.55) = 12.0% as important as the first factor. The third factor is about (0.472×2.08)/(3.804×4.77) = 5.4% as important as the second factor.
Chapter 9: Value at Risk

9.12. Suppose that each of two investments has a 4% chance of a loss of $10 million, a 2% chance of a loss of $1 million, and a 94% chance of a profit of $1 million. They are independent of each other.

(a) What is the VaR for one of the investments when the confidence level is 95%?
(b) What is the expected shortfall when the confidence level is 95%?
(c) What is the VaR for a portfolio consisting of the two investments when the confidence level is 95%?
(d) What is the expected shortfall for a portfolio consisting of the two investments when the confidence level is 95%?
(e) Show that, in this example, VaR does not satisfy the subadditivity condition whereas expected shortfall does.

(a) A loss of $1 million extends from the 94 percentile point of the loss distribution to the 96 percentile point. The 95% VaR is therefore $1 million.
(b) The expected shortfall for one of the investments is the expected loss conditional that the loss is in the 5 percent tail. Given that we are in the tail there is a 20% chance than the loss is $1 million and an 80% chance that the loss is $10 million. The expected loss is therefore $8.2 million. This is the expected shortfall.
(c) For a portfolio consisting of the two investments there is a 0.04 $\times$ 0.04 = 0.0016 chance that the loss is $20 million; there is a 2 $\times$ 0.04 $\times$ 0.02 = 0.0016 chance that the loss is $11 million; there is a 2 $\times$ 0.04 $\times$ 0.94 = 0.0752 chance that the loss is $9 million; there is a 0.02 $\times$ 0.02 = 0.0004 chance that the loss is $2 million; there is a 2 $\times$ 0.2 $\times$ 0.94 = 0.0376 chance that the loss is zero; there is a 0.94 $\times$ 0.94 = 0.8836 chance that the profit is $2 million. It follows that the 95% VaR is $9 million.
(d) The expected shortfall for the portfolio consisting of the two investments is the expected loss conditional that the loss is in the 5% tail. Given that we are in the tail, there is a 0.0016/0.05 = 0.032 chance of a loss of $20 million, a 0.0016/0.05 = 0.032 chance of a loss of $11 million; and a 0.936 chance of a loss of $9 million. The expected loss is therefore $9.416.
(e) VaR does not satisfy the subadditivity condition because $9 > 1 + 1$. However, expected shortfall does because $9.416 < 8.2 + 8.2$.

9.13. Suppose that daily changes for a portfolio have first-order correlation with correlation parameter 0.12. The 10-day VaR, calculated by multiplying the one-day VaR by $\sqrt{10}$, is $2$ million. What is a better estimate of the VaR that takes account of autocorrelation?

The correct multiplier for the variance is

$$10 + 2 \times 9 \times 0.12 + 2 \times 8 \times 0.12^2 + 2 \times 7 \times 0.12^3 + \ldots + 2 \times 0.12^9 = 10.417$$

The estimate of VaR should be increased to $2 \times \sqrt{10.417} / \sqrt{10} = 2.229$
9.14. Suppose that we back-test a VaR model using 1,000 days of data. The VaR confidence level is 99% and we observe 15 exceptions. Should we reject the model at the 5% confidence level? Use Kupiec’s two-tailed test.

In this case $p = 0.01$, $m = 15$, $n = 1000$. Kupiec’s test statistic is

$$-2 \ln[0.999^{0.01} \times 0.01^{15}] + 2 \ln[(1 - 15/1000)^{0.01} \times (15/1000)^{15}] = 2.19$$

This is less than 3.84. We should not therefore reject the model.
Chapter 10: Volatility

10.18. (Spreadsheet Provided)
Suppose that observations on a stock price (in dollars) at the end of each of 15 consecutive days are as follows:
30.2, 32.0, 31.1, 30.1, 30.2, 30.9, 30.5, 31.1, 31.3, 30.8, 30.3, 29.9, 29.8
Estimate the daily volatility using both approaches in Section 10.5?

The approach in equation (10.2) gives 2.28%. The approach in equation (10.4) gives 2.24%.

10.19.
Suppose that the price of an asset at close of trading yesterday was $300 and its volatility was estimated as 1.3% per day. The price at the close of trading today is $298. Update the volatility estimate using
(a) The EWMA model with \( \lambda = 0.94 \)
(b) The GARCH(1,1) model with \( \omega = 0.000002, \alpha = 0.04, \) and \( \beta = 0.94. \)

The proportional change in the price of the asset is \(-2/300 = -0.00667.\)
(a) Using the EWMA model the variance is updated to
\[
0.94 \times 0.013^2 + 0.06 \times 0.00667^2 = 0.0016153
\]
so that the new daily volatility is \(\sqrt{0.0016153} = 0.01271\) or 1.271% per day.
(b) Using GARCH (1,1) the variance is updated to
\[
0.000002 + 0.94 \times 0.013^2 + 0.04 \times 0.00667^2 = 0.0016264
\]
so that the new daily volatility is \(\sqrt{0.0016264} = 0.1275\) or 1.275% per day.

10.20. (Spreadsheet Provided)
An Excel spreadsheet containing over 900 days of daily data on a number of different exchange rates and stock indices can be downloaded from the author’s website: www.rotman.utoronto.ca/~hull/RMFI/data. Choose one exchange rate and one stock index. Estimate the value of \( \lambda \) in the EWMA model that minimizes the value of
\[
\sum (v_i - \beta_i)^2
\]
where \( v_i \) is the variance forecast made at the end of day \( i - 1 \) and \( \beta_i \) is the variance calculated from data between day \( i \) and day \( i + 25 \). Use the Solver tool in Excel. To start the EWMA calculations, set the variance forecast at the end of the first day equal to the square of the return on that day.

In the spreadsheet the first 25 observations on \( (v_i-\beta_i)^2 \) are ignored so that the results are not unduly influenced by the choice of starting values. The best values of \( \lambda \) for EUR, CAD, GBP and JPY were found to be 0.947, 0.898, 0.950, and 0.984, respectively. The best values of \( \lambda \) for S&P500, NASDAQ, FTSE100, and Nikkei225 were found to be 0.874, 0.901, 0.904, and 0.953, respectively.
10.21.
Suppose that the parameters in a GARCH(1,1) model are \( \alpha = 0.03, \beta = 0.95 \) and \( \omega = 0.000002 \).
(a) What is the long-run average volatility?
(b) If the current volatility is 1.5\% per day, what is your estimate of the volatility in 20, 40, and 60 days?
(c) What volatility should be used to price 20-, 40-, and 60-day options?
(d) Suppose that there is an event that increases the volatility from 1.5\% per day to 2\% per day. Estimate the effect on the volatility in 20, 40, and 60 days.
(e) Estimate by how much the event increases the volatilities used to price 20-, 40-, and 60-day options.

(a) The long-run average variance, \( \bar{V}_t \), is

\[
\omega \frac{1 - \alpha - \beta}{\omega} = \frac{0.000002}{0.02} = 0.0001
\]

The long run average volatility is \( \sqrt{0.0001} = 0.01 \) or 1\% per day.

(b) From equation (10.14) the expected variance in 20 days is
\[ 0.0001 + 0.98^{20} (0.015^2 - 0.0001) = 0.000183 \]

The expected volatility per day is therefore \( \sqrt{0.000183} = 0.0135 \) or 1.35\%. Similarly the expected volatilities in 40 and 60 days are 1.25\% and 1.17\%, respectively.

(c) In equation (10.15) \( a = \ln(1/0.98) = 0.0202 \). The variance used to price 20-day options is
\[
252 \left[ 0.0001 + \frac{1 - e^{-0.0202 \times 20}}{0.0202 \times 20} (0.015^2 - 0.0001) \right] = 0.051
\]

so that the volatility is 22.61\%. Similarly, the volatilities that should be used for 40- and 60-day options are 21.63\% and 20.85\% per annum, respectively.

(d) From equation (10.14) the expected variance in 20 days is
\[ 0.0001 + 0.98^{20} (0.02^2 - 0.0001) = 0.0003 \]

The expected volatility per day is therefore \( \sqrt{0.0003} = 0.0173 \) or 1.73\%. Similarly the expected volatilities in 40 and 60 days are 1.53\% and 1.38\% per day, respectively.

(e) When today’s volatility increases from 1.5\% per day (23.81\% per year) to 2\% per day (31.75\% per year) the equation (10.16) gives the 20-day volatility increase as
\[
\frac{1 - e^{-0.0202 \times 20}}{0.0202 \times 20} \times \frac{23.81}{22.61} \times (31.75 - 23.81) = 6.88
\]

or 6.88\% bringing the volatility up to 29.49\%. Similarly the 40- and 60-day volatilities increase to 27.37\% and 25.70\%.

10.22. (Spreadsheet Provided)
Estimate parameters for the EWMA and GARCH(1,1) model on the euro-USD exchange rate data between July 27, 2005, and July 27, 2010. This data can be found on the author’s website: www.rotman.utoronto.ca/~hull/RMFI/data

As the spreadsheets show the optimal value of \( \lambda \) in the EWMA model is 0.958 and the log likelihood objective function is 11,806.4767. In the GARCH (1,1) model, the optimal values of
$\omega$, $\alpha$, and $\beta$ are 0.0000001330, 0.04447, and 0.95343, respectively. The long-run average daily volatility is 0.7954% and the log likelihood objective function is 11,811.1955.

10.23.
The probability that the loss from a portfolio will be greater than $10$ million in one month is estimated to be 5%.
(a) What is the one-month 99% VaR assuming the change in value of the portfolio is normally distributed with zero mean?
(b) What is the one-month 99% VaR assuming that the power law applies with $a = 3$?

(a) The 99% VaR is

$$10 \times \frac{N^{-1}(0.99)}{N^{-1}(0.95)} = 14.14$$

or $14.14$ million.

(b) The probability that the loss is greater than $x$ is $Kx^\alpha$. We know that $\alpha = 3$ and $K \times 10^{-3} = 0.05$.
It follows that $K = 50$ and value of $x$ that is the 99% VaR is given by

$$50x^3 = 0.01$$

or

$$x = (5000)^{1/3} = 17.10$$

The 99% VaR using the power law is $17.10$ million.
**Chapter 11: Correlations and Copulas**

11.16. Suppose that the price of Asset X at close of trading yesterday was $300 and its volatility was estimated as 1.3% per day. The price of X at the close of trading today is $298. Suppose further that the price of Asset Y at the close of trading yesterday was $8, its volatility was estimated as 1.5% per day, and its correlation with X was estimated as 0.8. The price of Y at the close of trading today is unchanged at $8. Update the volatility of X and Y and the correlation between X and Y using

(a) The EWMA model with \( \lambda = 0.94 \)
(b) The GARCH(1,1) model with \( \omega = 0.000002, \alpha = 0.04, \text{ and } \beta = 0.94. \)

In practice, is the \( \omega \) parameter likely to be the same for X and Y?

The proportional change in the price of X is \(-2/300 = -0.00667\). Using the EWMA model the variance is updated to

\[
0.94 \times 0.013^2 + 0.06 \times 0.00667^2 = 0.00016153
\]

so that the new daily volatility is \(\sqrt{0.00016153} = 0.01271\) or 1.271% per day.

Using GARCH (1,1), the variance is updated to

\[
0.000002 + 0.94 \times 0.013^2 + 0.04 \times 0.00667^2 = 0.00016264
\]

so that the new daily volatility is \(\sqrt{0.00016264} = 0.1275\) or 1.275% per day.

The proportional change in the price of Y is zero. Using the EWMA model the variance is updated to

\[
0.94 \times 0.015^2 + 0.06 \times 0 = 0.0002115
\]

so that the new daily volatility is \(\sqrt{0.0002115} = 0.01454\) or 1.454% per day.

Using GARCH (1,1), the variance is updated to

\[
0.000002 + 0.94 \times 0.015^2 + 0.04 \times 0 = 0.0002135
\]

so that the new daily volatility is \(\sqrt{0.0002135} = 0.01461\) or 1.461% per day.

The initial covariance is \(0.8 \times 0.013 \times 0.015 = 0.000156\). Using EWMA the covariance is updated to

\[
0.94 \times 0.000156 + 0.06 \times 0 = 0.00014664
\]

so that the new correlation is \(0.00014664/(0.01454 \times 0.01271) = 0.7934\). Using GARCH (1,1) the covariance is updated to

\[
0.000002 + 0.94 \times 0.000156 + 0.04 \times 0 = 0.00014864
\]

so that the new correlation is \(0.00014864/(0.01461 \times 0.01275) = 0.7977\).

For a given \( \alpha \) and \( \beta \), the \( \omega \) parameter defines the long run average value of a variance or a covariance. There is no reason why we should expect the long run average daily variance for X and Y should be the same. There is also no reason why we should expect the long run average covariance between X and Y to be the same as the long run average variance of X or the long run variance of Y.
average variance of Y. In practice, therefore, we are likely to want to allow \( \omega \) in a GARCH(1,1) model to vary from market variable to market variable. (Some instructors may want to use this problem as a lead-in to multivariate GARCH models.)

11.17. **Spreadsheet Provided**

The probability density function for an exponential distribution is \( \lambda e^{-\lambda x} \) where \( x \) is the value of the variable and \( \lambda \) is a parameter. The cumulative probability distribution is \( 1 - e^{-\lambda x} \). Suppose that two variables \( V_1 \) and \( V_2 \) have exponential distributions with \( \lambda \) parameters of 1.0 and 2.0, respectively. Use a Gaussian copula to define the correlation structure between \( V_1 \) and \( V_2 \) with a copula correlation of \(-0.2\). Produce a table similar to Table 11.3 using values of 0.25, 0.5, 0.75, 1, 1.25, and 1.5 for \( V_1 \) and \( V_2 \). A spreadsheet for calculating the cumulative bivariate normal distribution is on the author’s website: www.rotman.utoronto.ca/~hull.

The probability that \( V_1 < 0.25 \) is \( 1 - e^{-1.0 \times 0.25} = 0.221 \). The probability that \( V_2 < 0.25 \) is \( 1 - e^{-2.0 \times 0.25} = 0.393 \). These are transformed to the normal variates \(-0.768\) and \(-0.270\). Using the Gaussian copula model the probability that \( V_1 < 0.25 \) and \( V_2 < 0.25 \) is \( M(-0.768, -0.270, -0.2) = 0.065 \). The other cumulative probabilities are shown in the table below and are calculated similarly.

<table>
<thead>
<tr>
<th>( V_1 )</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.065</td>
<td>0.117</td>
<td>0.153</td>
<td>0.177</td>
<td>0.193</td>
<td>0.203</td>
</tr>
<tr>
<td>0.50</td>
<td>0.125</td>
<td>0.219</td>
<td>0.282</td>
<td>0.323</td>
<td>0.349</td>
<td>0.366</td>
</tr>
<tr>
<td>0.75</td>
<td>0.177</td>
<td>0.303</td>
<td>0.386</td>
<td>0.439</td>
<td>0.472</td>
<td>0.493</td>
</tr>
<tr>
<td>1.00</td>
<td>0.219</td>
<td>0.371</td>
<td>0.469</td>
<td>0.531</td>
<td>0.569</td>
<td>0.593</td>
</tr>
<tr>
<td>1.25</td>
<td>0.254</td>
<td>0.426</td>
<td>0.535</td>
<td>0.603</td>
<td>0.645</td>
<td>0.672</td>
</tr>
<tr>
<td>1.50</td>
<td>0.282</td>
<td>0.469</td>
<td>0.587</td>
<td>0.660</td>
<td>0.705</td>
<td>0.733</td>
</tr>
</tbody>
</table>

11.18. **Spreadsheet Provided**

Create an Excel spreadsheet to produce a chart similar to Figure 11.5 showing samples from a bivariate Student t-distribution with four degrees of freedom where the correlation is 0.5. Next suppose that the marginal distributions of \( V_1 \) and \( V_2 \) are Student t with four degrees of freedom but that a Gaussian copula with a copula correlation parameter of 0.5 is used to define the correlation between the two variables. Construct a chart showing samples from the joint distribution. Compare the two charts you have produced.

The procedure for taking a random sample from a bivariate Student t-distribution is described on page 244. This can be used to produce Figure 11.5. For the second part of the question we
sample $U_1$ and $U_2$ from a bivariate normal distribution where the correlation is 0.5 as described in Section 11.3. We then convert each sample into a variable with a Student $t$-distribution on a percentile-to-percentile basis. Suppose that $U_1$ is in cell C1. The Excel function TINV gives a “two-tail” inverse of the $t$-distribution. An Excel instruction for determining $V_1$ is therefore
$$=IF(NORMSDIST(C1)<0.5,-TINV(2*NORMSDIST(C1),4),TINV(2*(1-NORMSDIST(C1)),4)).$$

The scatter plot shows that there is much less tail correlation when the normal copula is used for the $t$-distributions.

**11.19. (Spreadsheet Provided)**

Suppose that a bank has made a large number loans of a certain type. The one-year probability of default on each loan is 1.2%. The bank uses a Gaussian copula for time to default. It is interested in estimating a “99.97% worst case” for the percent of loan that default on the portfolio. Show how this varies with the copula correlation.

The WCDR with a 99.7% confidence level is from equation (10.12)
$$N\left(N^{-1}(0.012) + \sqrt{\rho}N^{-1}(0.9997)\right)$$
$$\sqrt{1-\rho}$$

The table below gives the variation of this with the copula correlation.

<table>
<thead>
<tr>
<th>Copula Correlation</th>
<th>WCDR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.2</td>
</tr>
<tr>
<td>0.1</td>
<td>10.8</td>
</tr>
<tr>
<td>0.2</td>
<td>21.0</td>
</tr>
<tr>
<td>0.3</td>
<td>32.6</td>
</tr>
<tr>
<td>0.4</td>
<td>45.5</td>
</tr>
<tr>
<td>0.5</td>
<td>59.5</td>
</tr>
<tr>
<td>0.6</td>
<td>73.7</td>
</tr>
<tr>
<td>0.7</td>
<td>86.9</td>
</tr>
<tr>
<td>0.8</td>
<td>96.5</td>
</tr>
<tr>
<td>0.9</td>
<td>99.9</td>
</tr>
</tbody>
</table>
11.20. (Spreadsheet Provided)

The default rates in the last 15 years for a certain category of loans is 2%, 4%, 7%, 12%, 6%, 5%, 8%, 14%, 10%, 2%, 3%, 2%, 6%, 7%, 9%. Use the maximum likelihood method to calculate the best fit values of the parameters in Vasicek’s model. What is the probability distribution of the default rate? What is the 99.9% worst case default rate?

The maximum likelihood estimates of $\rho$ and PD are 0.086 and 6.48%. The 99.9% worst case default rate is 26.18%. 
Chapter 12: Basel I, Basel II and Solvency II

12.19.  
Why is there an add-on amount in Basel I for derivatives transactions? “Basel I could be improved if the add-on amount for a derivatives transaction depended on the value of the transaction.” How would you argue this viewpoint?

The capital requirement is the current exposure plus an add-on amount multiplied by the counterparty risk weight multiplied by 8%. The add-on amount is to allow for a possibility that the exposure will increase prior to a default. To argue for a relationship between the add-on amount and the value of the transaction, consider two cases:
1. The value of the transaction is zero.
2. The value of the transaction is –$10 million

The current exposure is zero in both cases. In the first case any increase in the value of the transaction will lead to an exposure. In the second case the transaction has to increase in value by more than $10 million before there is an exposure—and it might be very unlikely that this will happen. However, the capital required is the same in both cases.

12.20.  
Estimate the capital required under Basel I for a bank that has the following transactions with another bank. Assume no netting.

(a) A two-year forward contract on a foreign currency, currently worth $2 million, to buy foreign currency worth $50 million
(b) A long position in a six-month option on the S&P 500. The principal is $20 million and the current value is $4 million.
(c) A two-year swap involving oil. The principal is $30 million and the current value of the swap is –$5 million.

What difference does it make if the netting amendment applies?

Using Table 12.2 the credit equivalent amounts (in millions of dollars) for the three transactions are
(a) 2 + 0.05 × 50 = 4.5
(b) 4 + 0.06 × 20 = 5.2
(c) 0.12 × 30 = 3.6

The total credit equivalent amount is 4.5+5.2+3.6 = 13.3. The risk weighted amount is 13.3 × 0.2 = 2.66. The capital required is 0.08 × 2.66 or $0.2126 million.

If netting applies, the current exposure after netting is in millions of dollars 2+4−5 = 1. The NRR is therefore 1/6 = 0.1667. The credit equivalent amount is in millions of dollars
1 + (0.4 + 0.6 × 0.1667)×(0.05 × 50 + 0.06 × 20 + 0.12 × 30) = 4.65

The risk weighted amount is 0.2×4.65 = 0.93 and the capital required is 0.08×0.93 = 0.0744. In this case the netting amendment reduces the capital by about 65%.
12.21.
A bank has the following transaction with a AA-rated corporation
(a) A two-year interest rate swap with a principal of $100 million that is worth $3 million
(b) A nine-month foreign exchange forward contract with a principal of $150 million that is worth $5 million
(c) An long position in a six-month option on gold with a principal of $50 million that is worth $7 million
What is the capital requirement under Basel I if there is no netting? What difference does it make if the netting amendment applies? What is the capital required under Basel II when the standardized approach is used?

Using Table 12.2 the credit equivalent amount under Basel I (in millions of dollars) for the three transactions are
(a) 3 + 0.005 × 100 = 3.5
(b) 0.01 × 150 = 1.5
(c) 7 + 0.01 × 50 = 7.5
The total credit equivalent amount is 3.5 + 1.5 + 7.5 = 12.5. Because the corporation has a risk weight of 50% for off-balance sheet items the risk weighted amount is 6.25. The capital required is 0.08 × 6.25 or $0.5 million.
If netting applies, the current exposure after netting is in millions of dollars 3−5+7 =5. The NRR is therefore 5/10 = 0.5. The credit equivalent amount is in millions of dollars
5 + (0.4 + 0.6 × 0.5)×(0.005 × 100 + 0.01 × 150 + 0.01 × 50) = 6.75
The risk weighted amount is 3.375 and the capital required is 0.08 × 3.375 = 0.27. In this case the netting amendment reduces the capital by 46%.
Under Basel II when the standardized approach is used the corporation has a risk weight of 20% and the capital required is $0.108 million.

12.22.
Suppose that the assets of a bank consist of $500 million of loans to BBB-rated corporations. The PD for the corporations is estimated as 0.3%. The average maturity is three years and the LGD is 60%. What is the total risk-weighted assets for credit risk under the Basel II advanced IRB approach? How much Tier 1 and Tier 2 capital is required? How does this compare with the capital required under the Basel II standardized approach and under Basel I?

Under the Basel II advanced IRB approach
\[ \rho = 0.12[1 + e^{-50 \times 0.003}] = 0.2233 \]
\[ b = [0.11852 - 0.05478 \times \ln(0.003)]^2 = 0.1907 \]
\[ MA = \frac{1 + (3.0 - 2.5) \times 0.1907}{1 - 1.5 \times 0.1907} = 1.53 \]

and
\[ WCDR = N \left[ \frac{N^{-1}(0.003 + \sqrt{0.2233N^{-1}(0.999)})}{\sqrt{1-0.2233}} \right] = 0.0720 \]
The RWA is
\[ 500 \times 0.6 \times (0.0720 - 0.003) \times 1.53 \times 12.5 = 397.13 \]
The total capital is 8% of this or $31.77 million. Half of this must be Tier I. Under both the Basel II standardized approach and under Basel I the risk weight is 100% and the total capital required is 8% of $500 or $40 million.
Chapter 13: Basel 2.5, Basel III, and Dodd-Frank

13.13
Explain one way that the Dodd–Frank Act is in conflict with (a) the Basel international regulations and (b) the regulations introduced by other national governments.

The Basel international regulations make extensive use of external ratings (e.g., from Moody’s, S&P, and Fitch). The Dodd-Frank Act does not allow external ratings to be used in regulation. The Volcker provision restricting proprietary trading by banks operating in the U.S. has not been adopted by most jurisdictions outside the U.S.

A bank has the following balance sheet

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>3</td>
<td>Retail Deposits (stable)</td>
<td>25</td>
</tr>
<tr>
<td>Treasury Bonds (&gt;1 year)</td>
<td>5</td>
<td>Retail Deposits (less stable)</td>
<td>15</td>
</tr>
<tr>
<td>Corporate Bonds Rated A</td>
<td>4</td>
<td>Wholesale Deposits</td>
<td>44</td>
</tr>
<tr>
<td>Residential Mortgages</td>
<td>18</td>
<td>Preferred Stock (&gt; 1 yr)</td>
<td>4</td>
</tr>
<tr>
<td>Small Business Loans (&lt;1 yr)</td>
<td>60</td>
<td>Tier 2 capital</td>
<td>3</td>
</tr>
<tr>
<td>Fixed Assets</td>
<td>10</td>
<td>Tier 1 Capital</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

(a) What is the Net Stable Funding Ratio?
(b) The bank decides to satisfy Basel III by raising more retail deposits and keeping the proceeds in Treasury bonds. What extra retail deposits need to be raised?

The amount of stable funding is

\[ 25 \times 0.9 + 15 \times 0.8 + 44 \times 0.5 + 16 \times 1.0 = 72.5 \]

The required stable funding is

\[ 3 \times 0 + 5 \times 0.05 + 4 \times 0.5 + 18 \times 0.65 + 60 \times 0.85 + 10 \times 1.0 = 74.95 \]

The net stable funding ratio is

\[ \frac{72.5}{74.95} = 0.967 \]

If \( X \) is the amount of retail deposits we require

\[ 72.5 + 0.9X = 74.95 + 0.05X \]

so that

\[ 0.85X = 2.45 \]

or \( X = 2.88 \)
Chapter 14: Market Risk VaR: The Historical Simulation Approach

14.12. Suppose that a one-day 97.5% VaR is estimated as $13 million from 2,000 observations. The one-day changes are approximately normal with mean zero and standard deviation $6 million. Estimate a 99% confidence interval for the VaR estimate.

The standard error is
\[
\frac{1}{f(q)} \sqrt{\frac{0.025 \times 0.975}{2000}}
\]
where \( f(q) \) is an estimate of the loss probability density at the VaR point. In this case the 0.975 point on the approximating normal distribution is \( \text{NORMINV}(0.975,0,6) = 11.76 \). \( f(q) \) is estimated as \( \text{NORMDIST}(11.76,0,6,\text{FALSE}) = 0.0097 \). The standard error is therefore
\[
\frac{1}{0.0097} \sqrt{\frac{0.025 \times 0.975}{2000}} = 0.358
\]

A 99% confidence interval for the VaR is \( 13 - 2.576 \times 0.358 \) to \( 13 + 2.576 \times 0.358 \) or 12.077 to 13.923.

14.13. (Spreadsheet Provided)
Suppose that the portfolio considered in Section 14.1 has (in $000s) 3,000 in DJIA, 3,000 in FTSE, 1,000 in CAC 40, and 3,000 in Nikkei 225. Use the spreadsheet on the author’s web site to calculate what difference this makes to
(a) The one-day 99% VaR that is calculated in Section 14.1.
(b) The one-day 99% VaR that is calculated using the weighting-of observations procedure in Section 14.3.
(c) The one-day 99% VaR that is calculated using the volatility-updating procedure in Section 14.3
(d) The one-day 99% VaR that is calculated using extreme value theory in Section 14.4.

(a) $230,785 (see Ranked Losses worksheet)
(b) $262,456 (see Ranked Losses worksheet with weights)
(c) $629,943 (see Ranked Losses Vol Adjusted Scenarios Worksheet)
(d) The values of \( \beta \) and \( \xi \) given by Solver are 44.94 and 0.306. The VaR with 99% confidence given by extreme value theory is $230,484
14.14. (Spreadsheet Provided)
Investigate the effect of applying extreme value theory to the volatility adjusted results in Section 14.3 with \( u = 350 \).

The values of \( \beta \) and \( \xi \) given by Solver are 95.24 and 0.285. The VaR with 99% confidence given by extreme value theory is $544,493.

14.15. (Spreadsheet Provided)
The “weighting-of-observations” procedure in Section 14.3 gives the one-day 99% VaR equal to $282,204. Use the spreadsheets on the author’s web site to calculate the one-day 99% VaR when the \( \lambda \) parameter in this procedure is changed from 0.995 to 0.985.

The worst scenario (number 494) now has a weight of 0.01371, which is more than 0.01. As a result the 99% VaR is the loss associated with this scenario or $477,841.

14.16. (Spreadsheet Provided)
The “volatility-updating” procedure in Section 14.3 gives the one-day 99% VaR equal to $602,968. Use the spreadsheets on the author’s web site to calculate the one-day 99% VaR when the \( \lambda \) parameter in this procedure is changed from 0.94 to 0.92.

The one-day 99% VaR changes to $660,553.

14.17 (Spreadsheet Provided)
Values for the NASDAQ composite index during the 1,500 days preceding March 10, 2006, can be downloaded from the author’s web site. Calculate the one-day 99% VaR on March 10, 2006, for a $10 million portfolio invested in the index using
(a) The basic historical simulation approach.
(b) The exponential weighting scheme in Section 14.3 with \( \lambda = 0.995 \).
(c) The volatility-updating procedure in Section 14.3 with \( \lambda = 0.94 \). (Assume that the volatility is initially equal to the standard deviation of daily returns calculated from the whole sample.)
(d) Extreme value theory with \( u = 300 \).
(e) A model where daily returns are assumed to be normally distributed. (Use both an equal weights and the EWMA approach with \( \lambda = 0.94 \) to estimate the standard deviation of daily returns.) Discuss the reasons for the differences between the results you get.

(a) The fifteenth worst daily change in the NASDAQ during the period considered is about –5.39% and the 1-day 99% VaR is $538,938. (See Ranked Losses worksheet)
(b) When weights are assigned to each day and the daily changes are listed from the worst to the best, the weights we see that the VaR is $229,401. (See Ranked Losses with Weights worksheet)
(c) In this case we use the EWMA updating scheme to update the variance and therefore the volatility. The volatility estimate for the day after March 10, 2006 is 0.755% per day. The volatility changes during the 1,500 day period as indicated in the chart below. Volatility was relatively low on March 10, 2006. Earlier observations are scaled down to reflect this. The resulting estimate of the 1-day 99% VaR is $160,985. (See Ranked Losses, Volatility Scaling worksheet)
(d) When extreme value theory with $u = 300,000$ (corresponding to a loss of 3%) is used in conjunction with the basic historical simulation VaR methodology, the VaR estimate changes from $538,938$ to $513,891$. (See Extreme Val Theory worksheet)
(e) The standard deviation of daily changes for the whole sample is 2.0123%. Assuming normality the 99% worst case daily change is $\text{NORMSINV}(0.99) \times 2.0123$ or about 4.68%. This gives a VaR estimate of about $468,000$. As indicated in c) the volatility estimate on March 10, 2006 when the EWMA method is used for updating is 0.755%. Assuming normality the 99% worst case daily change is $\text{NORMSINV}(0.99) \times 0.755$ or about 1.76%. This gives a VaR estimate of about $176,000$.

The main reason for the differences between the VaR estimates is indicated in the chart. The NASDAQ was a lot more volatile during the first part of the period considered. When the data are given equal weight we get a relatively high value of VaR. When weights decline exponentially large losses experienced over two years ago are given relatively little weight. When the volatility updating scheme in c) is used their impact is also considerably reduced because the losses are “scaled down.”
Chapter 15: Market Risk VaR: The Model-Building Approach

15.16. Consider a position consisting of a $300,000 investment in gold and a $500,000 investment in silver. Suppose that the daily volatilities of these two assets are 1.8% and 1.2% respectively, and that the coefficient of correlation between their returns is 0.6. What is the 10-day 97.5% VaR for the portfolio? By how much does diversification reduce the VaR?

The variance of the portfolio (in thousands of dollars) is

\[0.018^2 \times 300^2 + 0.012^2 \times 500^2 + 2 \times 300 \times 500 \times 0.6 \times 0.018 \times 0.012 = 104.04\]

The standard deviation is \(\sqrt{104.04} = 10.2\). Since \(N(-1.96) = 0.025\), the 1-day 97.5% VaR is 10.2 \(\times 1.96 = 19.99\) and the 10-day 97.5% VaR is \(\sqrt{10} \times 19.99 = 63.22\). The 10-day 97.5% value at risk for the gold investment is 5,400 \(\times 10 \times 1.96 = 33,470\). The 10-day 97.5% value at risk for the silver investment is 6,000 \(\times \sqrt{10} \times 1.96 = 37,188\). The diversification benefit is 33,470 + 37,188 − 63,220 = $7,438

15.17. Consider a portfolio of options on a single asset. Suppose that the delta of the portfolio is 12, the value of the asset is $10, and the daily volatility of the asset is 2%. Estimate the one-day 95% VaR for the portfolio from the delta.

An approximate relationship between the daily change in the value of the portfolio, \(\Delta P\) and the proportional daily change in the value of the asset \(\Delta x\) is

\[\Delta P = 10 \times 12 \Delta x = 120 \Delta x\]

The standard deviation of \(\Delta x\) is 0.02. It follows that the standard deviation of \(\Delta P\) is 2.4. The 1-day 95% VaR is 2.4 \(\times 1.65 = 3.96\).

15.18. Suppose that you know the gamma of the portfolio in Problem 15.17 is –2.6. Derive a quadratic relationship between the change in the portfolio value and the percentage change in the underlying asset price in one day.

(a) Calculate the first three moments of the change in the portfolio value.
(b) Using the first two moments and assuming that the change in the portfolio is normally distributed, calculate the one-day 95% VaR for the portfolio.
(c) Use the third moment and the Cornish–Fisher expansion to revise your answer to (b).

Using the same notation as in Problem 15.17, the quadratic relationship is

\[\Delta P = 10 \times 12 \Delta x + 0.5 \times 10^2 \times (-2.6)(\Delta x)^2\]

or

\[\Delta P = 120 \Delta x - 130(\Delta x)^2\]

(a) From page 339, the first three moments of \(\Delta P\) are \(-130\sigma^2\), \(120^2 \sigma^2 + 3 \times 130^2 \sigma^4\), and \(-9 \times 120^2 \times 130\sigma^4 - 15 \times 130^3 \sigma^6\) where \(\sigma\) is the standard deviation of \(\Delta x\). Substituting
\( \sigma = 0.02 \), the first three moments are \(-0.052, 5.768, \) and \(-2.698. \)

(b) The first two moments imply that the mean and standard deviation of \( \Delta P \) are \(-0.052 \) and \(2.402, \) respectively. The 5 percentile point of the distribution is \(-0.052 - 2.402 \times 1.65 = -4.02. \) The 1-day 95% VaR is therefore \( 4.02. \)

(c) The skewness of the distribution is

\[
\frac{1}{2.402^3} \left( -2.698 + 3 \times 5.768 \times 0.052 - 2 \times 0.052^2 \right) = -0.13
\]

We can use the Cornish-Fisher expansion in Section 13.7 setting \( q = 0.05 \) to obtain

\[
w_q = -1.65 - \frac{1}{6} (1.65^2 - 1) \times 0.13 = -1.687
\]

so that the 5 percentile point of the distribution is

\[-0.052 - 2.402 \times 1.687 = -4.10. \]

The 1-day 95% VaR is therefore \( 4.10. \)

15.19.

A company has a long position in a two-year bond and a three-year bond as well as a short position in a five-year bond. Each bond has a principal of \( \$100 \) and pays a 5% coupon annually. Calculate the company’s exposure to the one-year, two-year, three-year, four-year, and five-year rates. Use the data in Tables 8.7 and 8.8 to calculate a 20-day 95% VaR on the assumption that rate changes are explained by (a) one factor, (b) two factors, and (c) three factors. Assume that the zero-coupon yield curve is flat at 5%.

The cash flows are as follows

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-yr bond</td>
<td>5</td>
<td>105</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-yr bond</td>
<td>5</td>
<td>5</td>
<td>105</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-yr bond</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-105</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>105</td>
<td>100</td>
<td>-5</td>
<td>-105</td>
</tr>
<tr>
<td>Present Value</td>
<td>4.756</td>
<td>95.008</td>
<td>86.071</td>
<td>-4.094</td>
<td>-81.774</td>
</tr>
<tr>
<td>Impact of 1bp change</td>
<td>-0.0005</td>
<td>-0.0190</td>
<td>-0.0258</td>
<td>0.0016</td>
<td>0.0409</td>
</tr>
</tbody>
</table>

The duration relationship is used to calculate the last row of the table. When the one-year rate increases by one basis point, the value of the cash flow in year 1 decreases by \( 1 \times 0.0001 \times 4.756 = 0.0005; \) when the two year rate increases by one basis point, the value of the cash flow in year 2 decreases by \( 2 \times 0.0001 \times 95.008 = 0.0190; \) and so on.

The sensitivity to the first factor is

\[-0.0005 \times 0.216 - 0.0190 \times 0.331 - 0.0258 \times 0.372 + 0.0016 \times 0.392 + 0.0409 \times 0.404 \]

or \(-0.00116. \) Similarly the sensitivity to the second and third factors are \( 0.01589 \) and \(-0.01283. \)
Assuming one factor, the standard deviation of the one-day change in the portfolio value is 0.00116 \times 17.55 = 0.02043. The 20-day 95% VaR is therefore 0.0203 \times 1.645 \sqrt{20} = 0.149.

Assuming two factors, the standard deviation of the one-day change in the portfolio value is 
\sqrt{0.00116^2 \times 17.55^2 + 0.01589^2 \times 4.77^2} = 0.0785

The 20-day 95% VaR is therefore 0.0785 \times 1.645 \times 20 = 0.577.

Assuming three factors, the standard deviation of the one-day change in the portfolio value is 
\sqrt{0.00116^2 \times 17.55^2 + 0.01589^2 \times 4.77^2 + 0.01283^2 \times 2.08^2} = 0.610

The 20-day 95% VaR is therefore 0.0683 \times 1.645 \sqrt{20} = 0.502.

In this case the second has the most important impact on VaR.

15.20.

* A company has a position in bonds worth $6 million. The modified duration of the portfolio is 5.2 years. Assume that only parallel shifts in the yield curve can take place and that the standard deviation of the daily yield change (when yield is measured in percent) is 0.09. Use the duration model to estimate the 20-day 90% VaR for the portfolio. Explain carefully the weaknesses of this approach to calculating VaR. Explain two alternatives that give more accuracy.

The change in the value of the portfolio for a small change $\Delta y$ in the yield is approximately $-DB\Delta y$ where $D$ is the duration and $B$ is the value of the portfolio. It follows that the standard deviation of the daily change in the value of the bond portfolio equals $DB\sigma_y$ where $\sigma_y$ is the standard deviation of the daily change in the yield. In this case $D = 5.2$, $B = 6,000,000$, and $\sigma_y = 0.0009$ so that the standard deviation of the daily change in the value of the bond portfolio is

$5.2 \times 6,000,000 \times 0.0009 = 28,080$

The 20-day 90% VaR for the portfolio is $1.282 \times 28,080 \times \sqrt{20} = 160,990$ or $160,990$. This approach assumes that only parallel shifts in the term structure can take place. Equivalently it assumes that all rates are perfectly correlated or that only one factor drives term structure movements. Alternative more accurate approaches described in the chapter are (a) cash flow mapping and (b) a principal components analysis.

15.21. *(Spreadsheet Provided)*

* A bank has written European a call option on one stock and a European put option on another stock. For the first option, the stock price is 50, the strike price is 51, the volatility is 28% per annum, and the time to maturity is nine months. For the second option, the stock price is 20, the strike price is 19, the volatility is 25% per annum, and the time to maturity is one year. Neither stock pays a dividend, the risk-free rate is 6% per annum, and the correlation between stock price returns is 0.4. Calculate a 10-day 99% VaR

(a) Using only deltas.
(b) Using the partial simulation approach.
(c) Using the full simulation approach.

This assignment is useful for consolidating students’ understanding of alternative approaches to calculating VaR, but it is calculation intensive. Students should have some Excel (ideally VBA) skills. My answer assumes no VBA skills and follows the usual practice of assuming that the 10-
day 99% value at risk is $\sqrt{10}$ times the 1-day 99% value at risk. (Some students may try to calculate a 10-day VaR directly, which is fine.)

(a) From DerivaGem, the values of the two option positions are −5.413 and −1.014. The deltas are −0.589 and 0.284, respectively. An approximate linear model relating the change in the portfolio value to proportional change, $\Delta x_1$, in the first stock price and the proportional change, $\Delta x_2$, in the second stock price is

$$\Delta P = -0.589 \times 5 \Delta x_1 + 0.284 \times 20 \Delta x_2$$

or

$$\Delta P = -29.45 \Delta x_1 + 5.68 \Delta x_2$$

The daily volatility of the two stocks are $0.28/\sqrt{252} = 0.0176$ and $0.25/\sqrt{252} = 0.0157$, respectively. The one-day variance of $\Delta P$ is

$$29.45^2 \times 0.0176^2 + 5.68^2 \times 0.0157^2 - 2 \times 29.45 \times 0.0176 \times 5.68 \times 0.0157 \times 0.4 = 0.2407$$

The one day standard deviation is, therefore, 0.4906 and the 10-day 99% VaR is

$$2.33 \times 0.4906 = 3.61.$$ 

(b) In the partial simulation approach, we simulate changes in the stock prices over a one-day period (building in the correlation) and then use the quadratic approximation to calculate the change in the portfolio value on each simulation trial. Hitting F9 to run the simulation 20 times gives 20 different values for the 10-day 99% VaR. The results indicate that the 10-day 99% VaR is $3.83 \pm 0.04$.

(c) In the full simulation approach, we simulate changes in the stock price over one day (building in the underlying) and revalue the portfolio on each simulation trial. In this case the estimate of the 10-day 99% value at risk from running the simulation 20 times is $3.78 \pm 0.04$.

15.22.

A common complaint of risk managers is that the model-building approach (either linear or quadratic) does not work well when delta is close to zero. Test what happens when delta is close to zero in using Sample Application E in the DerivaGem Application Builder software. (You can do this by experimenting with different option positions and adjusting the position in the underlying to give a delta of zero.) Explain the results you get.

We can create a portfolio with zero delta in Sample Application E by changing the position in the stock from 1,000 to 513.58. (This reduces delta by 1,000−513.58 = 486.42.) In this case the true VaR is 48.86; the VaR given by the linear model is 0.00; and the VaR given by the quadratic model is -35.71.

Other zero-delta examples can be created by changing the option portfolio and then zeroing out delta by adjusting the position in the underlying asset. The results are similar. The software shows that neither the linear model nor the quadratic model gives good answers when delta is zero. The linear model always gives a VaR of zero because the model assumes that the portfolio has no risk. (For example, in the case of one underlying asset $\Delta P = Delta \times \Delta S$.) The quadratic model gives a negative VaR when gamma is positive because $\Delta P$ is then always positive: $\Delta P = 0.5 \Gamma (\Delta S)^2$. (Of course this would not be the case if the theta term were included.) In practice many portfolios do have deltas close to zero because of the hedging activities described in Chapter 7.
The problems referred to here have led many financial institutions to prefer historical simulation to the model building approach.

15.23. (Spreadsheet Provided)
The calculations in Section 15.3 assume that the investments in the DJIA, FTSE 100, CAC 40, and Nikkei 225 are $4 million, $3 million, $1 million, and $2 million, respectively. How does the VaR calculated change if the investments are $3 million, $3 million, $1 million, and $3 million, respectively? Carry out calculations when (a) volatilities and correlations are estimated using the equally weighted model and (b) when they are estimated using the EWMA model. What is the effect of changing \( \lambda \) from 0.94 to 0.90 in the EWMA calculations? Use the spreadsheets on the author’s web site.

(a) When the equally weighted model is used the worksheet shows that one-day 99% VaR is $215,007.
(b) When the EWMA model is used the worksheet shows that one-day 99% VaR is $447,404. Changing \( \lambda \) to 0.90 leads to a VaR of $500,403. This is higher because more recent (high) returns are given more weight.
Chapter 16: Credit Risk: Estimating Default Probabilities

16.23. (Spreadsheet Provided)

Suppose a three-year corporate bond provides a coupon of 7% per year payable semiannually and has a yield of 5% (expressed with semiannual compounding). The yields for all maturities on risk-free bonds is 4% per annum (expressed with semiannual compounding). Assume that defaults can take place every six months (immediately before a coupon payment) and the recovery rate is 45%. Estimate the default probabilities assuming a) the unconditional default probabilities are the same on each possible default date and b) assuming that the default probabilities conditional on no earlier default are the same on each possible default date.

(a) The market price of the bond is 105.51. The risk-free price is 108.40. The expected cost of defaults is therefore 2.89. The following table calculates the cost of defaults as 348.20Q where Q is the unconditional probability of default each six months. This must equal 2.89 so that Q must be 2.89/348.20 or 0.00831.

<table>
<thead>
<tr>
<th>Time (yrs)</th>
<th>Def. Prob.</th>
<th>Recovery Amount ($)</th>
<th>Risk-free Value ($)</th>
<th>Loss Given Default ($)</th>
<th>Discount Factor</th>
<th>PV of Expected Loss ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>Q</td>
<td>45</td>
<td>110.57</td>
<td>65.57</td>
<td>0.9804</td>
<td>64.28Q</td>
</tr>
<tr>
<td>1.0</td>
<td>Q</td>
<td>45</td>
<td>109.21</td>
<td>64.21</td>
<td>0.9612</td>
<td>61.73Q</td>
</tr>
<tr>
<td>1.5</td>
<td>Q</td>
<td>45</td>
<td>107.83</td>
<td>62.83</td>
<td>0.9423</td>
<td>59.20Q</td>
</tr>
<tr>
<td>2.0</td>
<td>Q</td>
<td>45</td>
<td>106.41</td>
<td>61.41</td>
<td>0.9238</td>
<td>56.74Q</td>
</tr>
<tr>
<td>2.5</td>
<td>Q</td>
<td>45</td>
<td>104.97</td>
<td>59.97</td>
<td>0.9057</td>
<td>54.32Q</td>
</tr>
<tr>
<td>3.0</td>
<td>Q</td>
<td>45</td>
<td>103.50</td>
<td>58.50</td>
<td>0.8880</td>
<td>51.95Q</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>348.20Q</td>
</tr>
</tbody>
</table>

(b) Suppose that Q’ is the default probability conditional on no earlier default. The unconditional default probabilities in 0.5, 1.0, 1.5, 2.0, 2.5, 3.0 years are Q’, Q’(1−Q’), Q’(1−Q’)², Q’(1−Q’)³, Q’(1−Q’)⁴, Q’(1−Q’)⁵. We must therefore find the value of Q’ that solves

\[ 64.28Q’ + 61.73Q’(1−Q’) + 59.20*Q’(1−Q’)² + 56.74Q’(1−Q’)³ \]
Using Solver in Excel we find that $Q^* = 0.00848$.

16.24.

A company has one-and two-year bonds outstanding, each providing a coupon of 8% per year payable annually. The yields on the bonds (expressed with continuous compounding are 6.0% and 6.6%, respectively. Risk-free rates are 4.5% for all maturities. The recovery rate is 35%. Defaults can take place halfway through each year. Estimate the risk-neutral default rate each year.

Consider the first bond. Its market price is $108e^{-0.06\times1} = 101.71$. Its default-free price is $108e^{-0.045\times0.5} = 103.25$. The present value of the loss from defaults is therefore 1.54. In this case losses can take place at only one time, halfway through the year. Suppose that the probability of default at this time is $Q_1$. The default-free value of the bond is $108e^{-0.045\times0.5} = 105.60$. The loss in the event of a default is $105.60 - 35 = 70.60$. The present value of the expected loss is $70.60e^{-0.045\times0.5}Q_1 = 69.03Q_1$. It follows that

$$69.03Q_1 = 1.54$$

so that $Q_1 = 0.0223$.

Now consider the second bond. It market price is 102.13 and its default-free value is 106.35. The present value of the loss from defaults is therefore 4.22. At time 0.5 the default free value of the bond is 108.77. The loss in the event of a default is therefore 73.77. The present value of the loss from defaults at this time is $72.13Q_2$ or 1.61. This means that the present value of the loss from defaults at the 1.5 year point is $4.22 - 1.61$ or 2.61. The default-free value of the bond at the 1.5 year point is 105.60. The loss in the event of a default is 70.60. The present value of the expected loss is $65.99Q_2$ where $Q_2$ is the probability of default in the second year. It follows that

$$65.99Q_2 = 2.61$$

so that $Q_2 = 0.0396$.

The probabilities of default in years one and two are therefore 2.23% and 3.96%.

16.25. (Spreadsheet Provided)

The value of a company’s equity is $4 million and the volatility of its equity is 60%. The debt that will have to be repaid in two years is $15 million. The risk-free interest rate is 6% per annum.

Use Merton’s model to estimate the expected loss from default, the probability of default, and the
recovery rate (as a percentage of the no-default value) in the event of default. Explain why Merton’s model gives a high recovery rate. (Hint: The Solver function in Excel can be used for this question.)

In this case $E_0 = 4$, $\sigma_E = 0.60$, $D = 15$, $r = 0.06$. Setting up the data in Excel, we can solve equations (14.4) and (14.5) by using the approach in footnote 23. The solution to the equations proves to be $V_0 = 17.084$ and $\sigma_V = 0.1576$. The probability of default is $N(-d_2)$ or 15.61%. The market value of the debt is $17.084 - 4 = 13.084$. The present value of the promised payment on the debt is $15e^{-0.06\times2} = 13.304$. The expected loss on the debt is, therefore, $(13.304 - 13.084)/13.304$ or 1.65% of its no-default value. The expected loss conditional on a default is $1.65/15.61$ or 11%. The recovery rate in the event of default is therefore 100–11 or 89%.

The reason the recovery rate is so high is as follows. There is a default if the value of the assets moves from 17.08 to below 15. A value for the assets significantly below 15 is unlikely. Conditional on a default, the expected value of the assets is, therefore, not hugely below 15. In practice, it is likely that companies manage to delay defaults until asset values are well below the face value of the debt.
Chapter 17: Counterparty Credit Risk in Derivatives

17.18. (Spreadsheet Provided)

Extend Example 17.2 so that default can happen in the middle of each month. Assume that the default probability during the first year is 0.001667 per month and the default probability during the second year is 0.0025 per month.

The CVA is 5.73.

17.19

Consider a European call option on a non-dividend-paying stock where the stock price is $52, the strike price $50, the risk-free rate is 5%, the volatility is 30%, and the time to maturity is one year. Answer the following questions assuming no recovery in the event of default, that the probability of default is independent of the option valuation, no collateral is posted, and no other transactions between the parties are outstanding.

(a) What is the value of the option assuming no possibility of a default?
(b) What is the value of the option to the buyer if there is a 2% chance that the option seller will default at maturity?
(c) Suppose that, instead of paying the option price up front, the option buyer agrees to pay the forward value of the option price at the end of option’s life. By how much does this reduce the cost of defaults to the option buyer in the case where there is a 2% chance of the option seller defaulting?
(d) If in case (c) the option buyer has a 1% chance of defaulting at the end of the life of the option, what is the default risk to the option seller? Discuss the two-sided nature of default risk in the case and the value of the option to each side.

(a) From DerivaGem the value is $8.41.
(b) This reduces the value of the option by 2% of $8.41, or $0.168, to $8.245 (see equation 15.2).
(c) The price paid for the option is $8.41e^{0.5}\times 1 = 8.845. In the event of a default a loss is made when the stock price is greater than 58.845 at maturity. The exposure is the price of a call with 58.845 as the strike price. The value of this call option is 4.64. The loss is 2% of this or $0.093, rather than $0.168.
(d) In the event that the buyer defaults the seller of the option loses when the stock price is less than 58.845 at maturity. The seller loses 8.845 when the stock price is less than 50 and 58.845 – S_T when it is between 50 and 58.845. The loss equals the price of a put with a strike price of 58.845 minus the price of a put with a strike price of 50. This is 8.616 – 3.975 = 4.641. The loss is 1% of this or 0.046. Theoretically, the present value of the price of the option in this case should be

8.41 – 0.093 + 0.046 = 8.37.

17.20

Suppose that the spread between the yield on a three-year riskless zero-coupon bond and a three-year zero-coupon bond issued by a bank is 210 basis points. The Black-Scholes–Merton
price of an option is $4.10. How much should you be prepared to pay for it if you buy it from a bank?

You should be prepared to pay $4.10 \times e^{-0.0210 \times 3} = $3.85. This assumes that the option will rank equally with the bond in the event of a default, no collateral is posted by the bank, and the option is not netted with any other derivatives transactions.

17.21. In Figure 17.3 where the CCP is used, suppose that an extra transaction between A and C which is worth 140 to A is cleared through the CCP. What effect does this have on the tables in figure 17.3.

In the case of the first table where all transactions are cleared bilaterally the exposures of A, B, and C become: 120, 100, and 0 for an average of 73.3. The second table becomes

<table>
<thead>
<tr>
<th>Dealer</th>
<th>Exposure after netting incl CCP</th>
<th>Exposure after netting excl CCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>170</td>
<td>50</td>
</tr>
<tr>
<td>B</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ave</td>
<td>96.7</td>
<td>56.7</td>
</tr>
</tbody>
</table>
Chapter 18: Credit Value at Risk

18.10. Explain carefully the distinction between real-world and risk-neutral default probabilities. Which is higher? A bank enters into a credit derivative where it agrees to pay $100 at the end of one year if a certain company’s credit rating falls from A to Baa or lower during the year. The one-year risk-free rate is 5%. Using Table 18.1, estimate a value for the derivative. What assumptions are you making? Do they tend to overstate or understate the value of the derivative?

Real world default probabilities are the true probabilities of defaults. They can be estimated from historical data. Risk-neutral default probabilities are the probabilities of default in a world where all market participants are risk neutral. They can be estimated from bond prices. Risk-neutral default probabilities are higher. This means that returns in the risk-neutral world are lower. From Table 15.1 the probability of a company moving from A to Baa or lower in one year is 6.24%. An estimate of the value of the derivative is therefore $0.0658 \times 100 \times e^{-0.05 \times 1} = 6.24$. The approximation in this is that we are using the real-world probability of a downgrade. To value the derivative correctly we should use the risk-neutral probability of a downgrade. Since the risk-neutral probability of a default is higher than the real-world probability, it seems likely that the same is true of a downgrade. This means that 6.24 is likely to be too low as an estimate of the value of the derivative.

18.11. Suppose that a bank has a total of $10 million of small exposures of a certain type. The one-year probability of default is 1% and the recovery rate averages 40%. Estimate the 99.5% one-year credit VaR using Vasicek’s model if the copula correlation parameter is 0.2.

The 99.5% worst case probability of default is

$$N\left(\frac{N^{-1}(0.01) + \sqrt{0.2}N^{-1}(0.995)}{\sqrt{0.8}}\right) = 0.0946$$

This gives the 99.5% credit VaR as $10 \times (1 - 0.4) \times 0.0946 = 0.568$ millions of dollars or $568,000.

18.12. (Spreadsheet Provided) Use the transition matrix in Table 18.1 and software on the author’s web site to calculate the transition matrix over 1.25 years.

The transition matrix is shown in the spreadsheet.

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Chapter 19: Scenario Analysis and Stress Testing

19.10. (Spreadsheet Provided)
What difference does it make to the worst-case scenario in Example 19.1 if (a) the options are American rather than European and (b) the options are barrier options that are knocked out if the asset price reaches $65? Use the DerivaGem Applications Builder in conjunction with Solver to search over asset prices between $40 and $60 and volatilities between 18% and 30%.

(a) In this case the worst outcome is an asset price of 42.86 and a volatility of 18%
(b) In this case the worst outcome is an asset price of 51.05 and a volatility of 19.45%

19.11.
What difference does it make to the VaR calculated in Example 19.2 if the exponentially weighted moving average model is used to assign weights to scenarios as described in Section 14.3?

The weights for the historical scenarios in Table 12.5 must be multiplied by 0.99. This leads to Table 17.1 being replaced by the table shown below. The VaR with a 99% confidence is $359,440.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Loss ($000s)</th>
<th>Probability</th>
<th>Cumul. Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>s5</td>
<td>850.000</td>
<td>0.00050</td>
<td>0.00050</td>
</tr>
<tr>
<td>s4</td>
<td>750.000</td>
<td>0.00050</td>
<td>0.00100</td>
</tr>
<tr>
<td>h494</td>
<td>499.395</td>
<td>0.00523</td>
<td>0.00623</td>
</tr>
<tr>
<td>s3</td>
<td>450.000</td>
<td>0.00200</td>
<td>0.00823</td>
</tr>
<tr>
<td>h339</td>
<td>359.440</td>
<td>0.00240</td>
<td>0.01063</td>
</tr>
<tr>
<td>h329</td>
<td>341.366</td>
<td>0.00229</td>
<td>0.01292</td>
</tr>
<tr>
<td>s2</td>
<td>300.000</td>
<td>0.00200</td>
<td>0.01492</td>
</tr>
<tr>
<td>h349</td>
<td>251.943</td>
<td>0.00253</td>
<td>0.01745</td>
</tr>
<tr>
<td>h487</td>
<td>247.571</td>
<td>0.00505</td>
<td>0.02250</td>
</tr>
<tr>
<td>h131</td>
<td>241.712</td>
<td>0.00085</td>
<td>0.02335</td>
</tr>
<tr>
<td>s1</td>
<td>235.000</td>
<td>0.00500</td>
<td>0.02835</td>
</tr>
<tr>
<td>h227</td>
<td>230.265</td>
<td>0.00137</td>
<td>0.02972</td>
</tr>
<tr>
<td>h495</td>
<td>227.332</td>
<td>0.00526</td>
<td>0.03498</td>
</tr>
<tr>
<td>h441</td>
<td>225.051</td>
<td>0.00401</td>
<td>0.03899</td>
</tr>
<tr>
<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
</tr>
</tbody>
</table>
Chapter 20: Operational Risk

20.12. Suppose that there is a 1% probability that operational risk losses of a certain type exceed $10 million. Use the power law to estimate the 99.97% worst-case operational risk loss when the _ parameter equals (a) 0.25, (b) 0.5, (c) 0.9, and (d) 1.0.

(a) In this case $K \times 10e^{-0.25} = 0.01$ so that $K = 0.01778$. The 99.97% worst case loss is (in millions of dollars) $x$ where $0.0889x^{-0.25} = 0.0003$. In this case $x = 12,345,679$.
(b) A similar calculation gives $x = 11,111$
(c) A similar calculation gives $x = 492$
(d) A similar calculation gives $x = 333$

20.13. Consider the following two events: (a) a bank loses $1 billion from an unexpected lawsuit relating to its transactions with a counterparty and (b) an insurance company loses $1 billion because of an unexpected hurricane in Texas. Suppose that you have the same investment in shares issued by both the bank and the insurance company. Which loss are you more concerned about? Why?

You should be more concerned about the bank loss. Most other insurance companies are likely to have suffered from the same type of loss as the insurance company in question and insurance premiums are likely to rise as a result. The bank’s loss is likely to be unique to the bank and in a competitive market the bank will be unable to recoup the loss from higher fees or higher net interest margins.

20.14. (Spreadsheet Provided)
The worksheet used to produce Figure 20.2 is on the author’s web site. How does the loss distribution change when the loss severity has a beta distribution with upper bound of 5, lower bound of zero, and the other parameters both 1?

The spreadsheet used to calculate Figure 18.2 is in the software section on my web site. In the instructions that are in cells B7:K1006 it is necessary to change EXP($D$2+NORMSINV(RAND())*$D$3) to BETAINV(RAND(),1,1,0,5). The mean loss increases to about 7.5 and the standard deviation to about 5.
Chapter 21: Liquidity Risk

21.13. Discuss whether hedge funds are good or bad for the liquidity of markets.

As discussed in the chapter, the major source of liquidity risk is liquidity black holes where most market participants want to be on the same side of the market at the same time. Hedge funds are not regulated in the same way as other financial institutions. It is therefore possible that they improve liquidity because they want to do different trades from other financial institutions. However, hedge funds are themselves big players in some markets. Hedge funds tend to follow similar trading strategies to each other. As a result, they respond in the same way to market events and can create (or make worse) black holes. (The LTCM situation in 1998 provides an example here.) Also hedge funds are reliant on leverage. When prime brokers cut their lines of credit they all tend to have to close out positions at the same time. (This happened in 2007.)

21.14. Suppose that a trader has bought some illiquid shares. In particular, the trader has 100 shares of A, which is bid $50 and offer $60, and 200 shares of B, which is bid $25 offer $35. What are the proportional bid–offer spreads? What is the impact of the high bid–offer spreads on the amount it would cost the trader to unwind the portfolio. If the bid–offer spreads are normally distributed with mean $10 and standard deviation $3, what is the 99% worst-case cost of unwinding in the future as a percentage of the value of the portfolio?

The proportional bid-offer spreads for share A and B are 10/55 =0.1818 and 10/30=0.3333. The mid-market values of the positions are $5,500 and $6,000, respectively. The cost, associated with bid-offer spreads, when the portfolio is unwound is

\[0.5 \times 0.1818 \times 5,500 + 0.5 \times 0.3333 \times 6,000 = 1,500\]

or $1,500. The standard deviation of the proportional bid offer spreads are 3/55=0.054545 and 3/30=0.1. The 99% worst case cost of unwinding is

\[0.5 \times (0.1818 + 2.326 \times 0.054545) \times 5,500 + 0.5 \times (0.3333 + 2.326 \times 0.1) \times 6,000 = 2547\]

or $2,547.

21.15. (Spreadsheet Provided)

A trader wishes to unwind a position of 200,000 units in an asset over eight days. The dollar bid–offer spread, as a function of daily trading volume \(q\), is \(a + b \times q\) where \(a = 0.2, b = 0.15\) and \(c = 0.1\) and \(q\) is measured in thousands. The standard deviation of the price change per day is $1.50. What is the optimal trading strategy for minimizing the 99% confidence level for the costs? What is the average time the trader waits before selling? How does this average time change as the confidence level changes?

The spreadsheet shows that the optimal trading strategy is to trade 33.4, 31.1, 28.7, 26.2, 23.5, 20.9, 18.7, and 17.4 on successive days. The average time until selling is 4.00 days. For confidence levels of 90%, 95%, 99% 99.9% and 99.99%, the average times until selling are 4.20, 4.13, 4.00, 3.87, and 3.78 days.
Chapter 22: Model Risk

22.13. Suppose that all options traders decide to switch from Black–Scholes to another model that makes different assumptions about the behavior of asset prices. What effect do you think this would have on (a) the pricing of standard options and (b) the hedging of standard options?

(a) As explained in the chapter the Black–Scholes model is used as an interpolation tool. It is likely that the shape of the volatility smile would change, but prices would change very little. A similar interpolation tool would emerge.

(b) Hedging would change as a different model would give different Greek letters.

22.14. Using Table 22.1, calculate the volatility a trader would use for an 11-month option with a strike price of 0.98.

Interpolation gives the volatility for a six-month option with a strike price of 0.98 as 12.82%. Interpolation also gives the volatility for a 12-month option with a strike price of 0.98 as 13.7%. A final interpolation gives the volatility of an 11-month option with a strike price of 0.98 as 13.55%. The same answer is obtained if the sequence in which the interpolations are done is reversed.

22.15. Suppose that a financial institution uses an imprecise model for pricing and hedging a particular type of structured product. Discuss how, if at all, it is likely to realize its mistake.

Suppose that a bank’s price for the product is too high. If other market participants are using a better model it is likely to realize its mistake when a) it gets lots of offers from other market participants to sell the product to the bank or b) when it tries to unwind all or part of its position. If it has no interaction on pricing with the rest of the market or if the rest of the market is pricing the product in the same way, it might never realize its mistake. In theory, its hedging should lead to losses, but in practice the losses are likely to be difficult to detect because hedging is never perfect. The mispricing is likely to be especially difficult to detect in the case of a highly structured product that is traded relatively infrequently and lasts several years.

22.16. A futures price is currently at $40. The risk-free interest rate is 5%. Some news is expected tomorrow that will cause the volatility over the next three months to be either 10% or 30%. There is a 60% chance of the first outcome and a 40% chance of the second outcome. Use the DerivaGem software to calculate a volatility smile for three-month options.

The calculations are shown in the following table. For example, when the strike price is 34, the price of a call option with a volatility of 10% is 5.926, and the price of 40 a call option when the volatility is 30% is 6.312. When there is a 60% chance of the first volatility and 40% of the...
second, the price is $0.6 \times 5.926 + 0.4 \times 6.312 = 6.080$. The implied volatility given by this price is 23.21. The table shows that the uncertainty about volatility leads to a classic volatility smile similar to that in Figure 20.2 of the text. In general when volatility is stochastic with the stock price and volatility uncorrelated we get a pattern of implied volatilities similar to that in Figure 20.2 of the text.

<table>
<thead>
<tr>
<th>Strike price</th>
<th>Call Price (10% Vol)</th>
<th>Call price (30% Vol)</th>
<th>Weighted Price</th>
<th>Implied Vol</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>5.926</td>
<td>6.312</td>
<td>6.080</td>
<td>23.21</td>
</tr>
<tr>
<td>36</td>
<td>3.962</td>
<td>4.749</td>
<td>4.277</td>
<td>21.03</td>
</tr>
<tr>
<td>38</td>
<td>2.128</td>
<td>3.423</td>
<td>2.646</td>
<td>18.88</td>
</tr>
<tr>
<td>40</td>
<td>0.788</td>
<td>2.362</td>
<td>1.418</td>
<td>18.00</td>
</tr>
<tr>
<td>42</td>
<td>0.177</td>
<td>1.560</td>
<td>0.730</td>
<td>18.80</td>
</tr>
<tr>
<td>44</td>
<td>0.023</td>
<td>0.988</td>
<td>0.409</td>
<td>20.61</td>
</tr>
<tr>
<td>46</td>
<td>0.002</td>
<td>0.601</td>
<td>0.242</td>
<td>22.43</td>
</tr>
</tbody>
</table>
Chapter 23: Economic Capital and RAROC

23.10. Suppose that daily gains (losses) are normally distributed with standard deviation of $5 million. 
(a) Estimate the minimum regulatory capital the bank is required to hold. (Assume a multiplicative factor of 4.0.) 
(b) Estimate the economic capital using a one-year time horizon and a 99.9% confidence level assuming that there is a correlation of 0.05 between gains (losses) on successive days.

(a) The 99% one-day VaR is $2.326 \times 5 = 11.63$. The 10-day VaR is $\sqrt{10}$ times this or 36.78. The regulatory capital is 4 times this or 147.13.
(b) As shown in Chapter 8, the autocorrelation of 0.05 leads to the ratio of the 252-day VaR to the one-day VaR being 
\[
\sqrt{N + 2(N - 1)\rho + 2(N - 2)\rho^2 \cdots 2\rho^{N-1}}
\]
where $N$ is 252 and $\rho$ is 0.05. This is 16.69. The economic capital is therefore 
\[3.09 \times 5 \times 16.69 = 257.81\]

23.11. Suppose that the economic capital estimates for two business units are

<table>
<thead>
<tr>
<th>Business Units</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Risk</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>Credit Risk</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Operational Risk</td>
<td>50</td>
<td>10</td>
</tr>
</tbody>
</table>

The correlation between market risk and credit risk in the same business unit is 0.3. The correlation between credit risk in one business unit and credit risk in another is 0.7. The correlation between market risk in one business unit and market risk in the other is 0.2. All other correlations are zero. Calculate the total economic capital. How much should be allocated to each business unit?

The economic capital is the square root of 
\[
10^2 + 50^2 + 30^2 + 30^2 + 50^2 + 10^2 + 2 \times 10 \times 50 \times 0.2 + 2 \times 30 \times 30 \times 0.7 \\
+ 2 \times 10 \times 30 \times 0.3 + 2 \times 50 \times 30 \times 0.3 
\]
or $97.673$.

When Business Unit 1 is increased by 1% the economic capital increases by 0.452 to 98.125.
The amount of capital that should be allocated to Business Unit 1 is therefore $0.452 / 0.01 = 45.2$.
When Business Unit 2 is increased by 1% the economic capital increases by 0.526 to 98.199.
The amount of capital that should be allocated to Business Unit 1 is therefore $0.526 / 0.01 = 52.6$.
Not that the total of the economic capital allocated is nearly exactly equal to the original economic capital. If the partial derivatives had been calculated exactly we would have exact equality, as shown by Euler’s theorem.
23.12. Suppose that a bank’s sole business is to lend in two regions of the world. The lending in each region has the same characteristics as in Example 23.5 of Section 23.8. Lending to Region A is three times as great as lending to Region B. The correlation between loan losses in the two regions is 0.4. Estimate the total RAROC.

Suppose that the lending to Region A is $3X$ and that to Region B is $X$. The economic capital for Region A is $0.04 \times 3X = 0.12X$. The economic capital for Region B is $0.04X$. The total economic capital using the hybrid approach is

$$\sqrt{(0.12X)^2 + (0.04X)^2 + 2 \times 0.4 \times 0.12X \times 0.04X} = 0.141X$$

The expected profit is 0.8% of $4X$ or $0.032X$. The RAROC is therefore

$$\frac{0.032X}{0.141X} = 0.227$$

or 22.7%.