Financial Planning with Multiple Objectives
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Financial Planning
with Multiple Objectives

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In recent years, there has been much interest in models of multiobjective decision-making. The finance literature reflects this with the formulation of problems in capital budgeting [6, 8, 13], working capital management [10], portfolio selection [7, ch. 9], and commercial bank management [2, 4, 5, 9, 11] as multiobjective decision-making problems. Given multiple conflicting objectives, resolution of any planning problem depends on subjective judgments and relative preferences of decision-makers. Unfortunately, how to include managerial preferences in normative decision models, and the effect of these preferences on optimal solutions, has scarcely been explored. Moreover, there has been little if any research on the relative merits of various programming techniques in dealing with multiobjective problems.

The purposes of this paper are to discuss introducing the subjective preferences of decision-makers into multiobjective programming models and to compare the characteristics of goal programming and multiobjective linear programming in dealing with multiobjective problems.

Multiobjective Problems and Programming Techniques

The assumption that firms pursue the single objective of wealth maximization rather than multiple objectives has been questioned in the literature. Even though some managers may seek this single objective, as writers such as Cyert and March [3] and Williamson [14] point out, others are more likely to be influenced by a mixture of operational and personal objectives. Possible objectives are profits, sales, stock price, growth, risk, liquidity, and market share, among others.

The introduction of the managerial utility function, and thus the subjective preferences of management for various objectives, constitutes the major conceptual problem in multiobjective decision-making.

Within a multiobjective framework, the firm’s decision-making process may be conveniently viewed as a utility-maximizing problem with the utility function being managerial in nature (see Becker [1]). To specify a multiobjective model, assume that a
decision-maker faces a vector of objectives, \( y \), which can be achieved through a vector of decision variables, \( x \), where \( y = Bx \) (\( B \) is a matrix of constants relating the decision variables to the objectives). Such a utility function may be specified, in general form, as follows:

\[
U = U(y_1, y_2, y_3, \ldots, y_n). \tag{1}
\]

The ability to achieve these objectives and thus maximize utility is constrained by a set of environmental conditions within which the decision-maker must operate. These include accounting constraints such as those imposed by the balance sheet and income statement, market constraints reflecting the extent of the demand for the firm's output, budget constraints resulting from capital rationing, and regulatory constraints that restrict the firm's activities. Let these constraints be denoted by \( Cx < z \) where \( z \) is a vector of constraint values and \( C \) is a matrix of constants relating the decision variables to the constraints.

The model presented allows two possible approaches to arriving at the utility-maximizing solution. First, the utility function can be specified in explicit functional form and the function then directly maximized in terms of the decision variables subject to the relevant constraints. This theoretically useful approach is found frequently in the academic finance literature. Its practical usefulness is highly uncertain, however, because the exact form of the utility function is not generally known even to decision-makers, let alone to the model builder or analyst. In addition, the utility function may not be stable. All that is known with reasonable certainty about the function are its arguments and the direction of change in utility given changes in these arguments.

A second alternative is to maximize utility indirectly by exploring the set of efficient solutions where the efficient set is determined by the constraints \( Cx \leq z \). Knowing the efficient set of solutions allows the decision-maker to avoid the problem of lack of specific information about the utility function. Instead, from the efficient set of solutions, given the tradeoffs that are implied by the efficient set, the decision-maker can select an optimal solution based only on his intuitive knowledge of the utility function.

**Goal Programming**

Goal programming allows the explicit introduction of multiple objectives into a programming model. To formulate the above problem as a goal programming model, the decision-maker first chooses desired values for each objective included in the problem. Next he determines the relative importance of deviations above and/or below these desired goal values. Finally, he assigns weights to these deviations that reflect his priorities.

The objective of the GP model is simultaneously to minimize the weighted sum of the absolute value of the deviations from prestated goal values for the objectives. The goal programming model can be formulated, in terms of the decision variables, as follows:

\[
\text{Minimize: } \sum_{i=1}^{n} w_i d_i + w^+_i d^+_i \\
\text{Subject to: } b_1 x + d_1 - d^+_1 = y^*_1 \\
\quad \vdots \\
\quad b_n x + d_n - d^+_n = y^*_n \\
C x \leq z \\
x \geq 0 \\
d_i, d^+_i \geq 0
\]

where \( b_i \) is the \( i \)th row of the matrix \( B \) introduced above, \( y^*_i \) is the desired value for the \( i \)th objective, \( w_i \) and \( w^+_i \) are the priority weights attached to underachievement and overachievement of the \( i \)th goal value, respectively, and \( d_i \) and \( d^+_i \) are the actual deviations below and above the \( i \)th goal value.

Given the utility function in Equation (1), the subjective preferences of management are included in the GP model through the goal values, \( y^*_i \), and the weights, \( w_i \). Since this requires a priori specification of certain aspects of the utility function, GP is closely associated with the direct utility maximization approach to multiobjective problems. This creates two problems. First, since utility functions are generally unknown, it may be impossible to select accurate values for the goals and weights. Second, since the goals and weights must be specified before the feasible set of solutions given by the constraints \( C x \leq z \) is known, GP may lead to solutions that are not efficient (see Zeleny and Cochrane [16]).

Although GP does allow explicit consideration of multiple objectives and the tradeoffs among these objectives, the solutions depend on the goal values and relative weights that are selected. Writers who have used goal programming, such as Sartoris and Spruill [10, p. 70], recognize the problem of accurately selecting goal and weight values when they say that
decision-makers may not have a “feel” for the appropriate goal and weight values. It should be apparent that this lack of “feel” is a direct result of lack of explicit knowledge of the utility function itself. They recommend in such cases that the GP model be solved several times using different weights and goals, thus in effect recommending exploring the efficient set of solutions.

**Multiobjective Linear Programming**

The formulation of a multiobjective linear programming problem (MOBLP) does not require the decision-maker to choose desired values for the objectives or *a priori* to determine relative weighting. All that is required is to state each objective as a linear function of the decision variables. The problem presented earlier can be formulated as an MOBLP problem as follows:

**Optimize:**

\[ y_1 = b_1x \]
\[ y_2 = b_2x \]
\[ \vdots \]
\[ y_n = b_nx \]

**Subject to:**

\[ Cx \leq z \]
\[ x \geq 0 \]

where \( b_i \) is the \( i \)th row of the matrix \( B \).

Unlike GP, multiobjective linear programming does not require introducing utility parameters into the decision-making process through the model itself. Instead, the MOBLP problem is specified simultaneously to maximize desirable objectives and to minimize undesirable objectives subject to the environmental constraints \( Cx \leq z \). Since no single solution is likely to optimize all objectives simultaneously, the MOBLP procedure locates a set of nondominated or efficient solutions to the problem. A nondominated solution is one in which you cannot achieve more of one objective without getting less of another. If more than one nondominated solution exists, which is generally the case for a multiobjective problem, the decision-maker faces a set of possible feasible solutions representing a range of values for the objectives. (In fact, one of the difficulties of MOBLP applications is that many problems yield such a large number of solutions that it may be difficult to deal with all information. It is possible to reduce this problem by using an algorithm similar to the one developed by Steuer [12], which reduces the total number of solutions through a screening process.)

In addition, a comparison of the various nondominated solutions will yield explicit measures of the tradeoffs between the objectives. Decision-makers, faced with the set of efficient solutions, may then subjectively choose the “best” solution based on their own subjective preferences for the objectives and the explicit tradeoffs that are required.

**A Bank Financial Planning Model and Numerical Example**

The following simplified balance sheet optimization model for a commercial bank demonstrates how multiobjective models are formulated as GP and MOBLP problems and compares the solutions obtained with each method. For expository purposes, the model is considerably simplified; asset categories are aggregated, and the model considers only a single period. Greater detail and multiple time periods will be more useful in practical applications. As the simplified model does contain the essential elements of a working model, it could be expanded to include greater disaggregation, multiple time periods, and additional objectives.

**Model Objectives**

Bank managers are assumed to pursue the objectives of profit and solvency. The decision variables to achieve these objectives are six types of...
assets categorized according to their liquidity classification as specified by the "Form for Analyzing Bank Capital" used by the Federal Reserve System. Exhibit 1 presents a list of the variables used and the notation employed. To specify the model, measures of these objectives must be developed. The objectives, stated in terms of the decision variables, are:

**Profit Function.** The specification of the profit function is straightforward. Exhibit 1 presents the assumed rates of return on the six categories of assets. Given that information, the profit function is

\[
\text{Profit} = 0.04X_2 + 0.045X_3 + 0.05X_4 + 0.06X_5 + 0.10X_6.
\]

(2)

**Solvency Measures.** The specification of measures of solvency is more difficult than that of profitability. Since the primary goals of bank managers, other than profitability, are generally stated in terms of liquidity and risk, measures of liquidity and risk would seem to accurately reflect the bank’s solvency considerations. For purposes of the model presented here, liquidity and risk are measured jointly by two related ratios: 1) the capital adequacy ratio employed by bank examiners of the Federal Reserve System, and 2) the risk asset to capital ratio.

The capital adequacy ratio is a comprehensive measure of the bank’s liquidity and risk. The explicit measure of capital adequacy employed here is the ratio of required to actual bank capital (hereafter referred to as the CA ratio). Banks have considerable discretion in determining this ratio and may choose to be less liquid and bear more risk, or vice versa. Since a full derivation of the capital adequacy function can be found elsewhere [2, pp. 56-61], it is not presented here. The capital adequacy or liquidity function is

\[
CA \cdot K = 0.005X_2 + 0.04X_3 + 0.40X_4 + 0.06X_5 + 0.10X_6 + 0.065L_1 + 0.04L_2 + 0.095L_3
\]

where CA is the ratio of required capital to actual bank capital (K), and L_1, L_2, and L_3 are auxiliary variables required for the piecewise linear formulation of the capital adequacy requirement. In order to state the capital adequacy equation in a form that can be used in the present model, both sides are divided by K, and the function becomes

\[
CA = 0.001X_2 + 0.008X_3 + 0.008X_4 + 0.012X_5 + 0.02X_6 + 0.013L_1 + 0.008L_2 + 0.019L_3.
\]

(3)

Since liquidity diminishes and risk increases as the CA ratio increases, banks can increase liquidity and decrease risk by decreasing the CA ratio.

A second objective reflecting the bank’s solvency is the risk asset to capital ratio (RA ratio). Risk assets are the least liquid assets held by banks; they have the highest risk of default. The RA ratio is a type of capital adequacy measure, and, as such, is related to and included in the more comprehensive measure of capital adequacy presented above. It is reasonable to believe, however, that the CA ratio does not totally reflect all the relevant solvency considerations, so the RA ratio may provide additional information on the bank’s liquidity and risk position. For instance, the rate of loan default tends to follow the business cycle, adding a considerable element of nondiversifiable risk. Therefore, bank managers may wish to maintain some direct control over the ratio of risk assets to capital which may not necessarily be reflected in the CA ratio.

To formulate the risk function, note that the asset category X_6 is assumed to be composed of risk assets. Thus, the risk function can be stated as follows:

\[
RA \cdot K = X_6
\]

where RA is the ratio of risk assets to capital. Again, in order to specify the risk function in usable form, both sides are divided by K, and the risk function becomes

\[
RA = 0.20X_6
\]

(4)

The bank is assumed to incur greater risk as the RA ratio increases.

**Constraints**

The model is also assumed to be subject to a number of constraints:

---

**Exhibit 1.** Data and Abbreviations Used for the Numerical Problem

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Rates of Return</th>
<th>Demand Deposits</th>
<th>Time Deposits</th>
<th>Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xi</td>
<td>0%</td>
<td>$40 million</td>
<td>$20 million</td>
<td>$5 million</td>
</tr>
<tr>
<td>X2</td>
<td>4%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X3</td>
<td>4.5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X4</td>
<td>5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X5</td>
<td>6%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X6</td>
<td>10%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K1</td>
<td>0.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K2</td>
<td>0.04</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>XI</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>X6</th>
<th>DD</th>
<th>TD</th>
<th>K</th>
<th>K1</th>
<th>K2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>4%</td>
<td>4.5%</td>
<td>5%</td>
<td>6%</td>
<td>10%</td>
<td>40 million</td>
<td>20 million</td>
<td>$5 million</td>
<td>0.14</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Capital Adequacy Constraints. The constraints imposed by the bank examiners' capital adequacy requirements are:

\[ X_1 + .995X_2 + .96X_3 + L_1 \geq .47DD + .36TD \]  
\[ X_1 + .995X_2 + .96X_3 + .9X_4 + L_2 \geq .47DD + .36TD \]  
\[ X_1 + .995X_2 + .96X_3 + .85X_4 + L_3 \geq .47DD + .36TD. \]

Diversification Constraints. Bank managers are assumed to place discretionary constraints on each earning asset category so that no category will be less than 5% of total liabilities plus capital. This diversification constraint can be stated as

\[ X_i \geq .05 (DD + TD + K); i = 2-6. \]

Required Reserves Constraint. Given the reserve requirements for demand and time deposits shown in Exhibit 1, the required reserves constraint is

\[ X_1 \geq .14DD + .04TD. \]

Balance Sheet Constraint. Finally, bank decisions are limited by the balance sheet constraint:

\[ \sum_{i=1}^{6} X_i = DD + TD + K. \]

Formulation of the Model as a GP and MOBLP Problem

The multiobjective bank management model presented above can be formulated as either a GP or MOBLP problem. To specify the model as a GP problem, management must select desired values for the objectives in Equations (2), (3), and (4). Once these values are selected, respecified versions of these equations become constraints for the GP problem. These constraints can be stated as:

\[ .04X_2 + .045X_3 + .05X_4 + .06X_5 + .10X_6 \]
\[ + d_{-1} - d_{+1} = \text{Profit Goal} \]  
\[ .001X_2 + .008X_3 + .008X_4 + .012X_5 + .02X_6 \]
\[ + .013L_1 + .008L_2 + .019L_3 + d_2 - d_{-2} = \text{CA Goal} \]  
\[ .2X_6 + d_3 - d_{-3} = \text{RA Goal}. \]

where \( d_{-1} \) and \( d_{+1} \) are the deviations below and above the goal value for the ith objective, respectively. The GP objective function is:

\[ Z = w_1d_1 + w_2d_2 + w_3d_3 + w_4d_4 + w_5d_5 + w_6d_6 \]

where \( w_i \) and \( w_{+1} \) are the priority weights selected by management to reflect the relative importance of underachievement and overachievement of the goal value for the ith objective function, respectively. Equation (14) is minimized subject to constraints (11) through (13), (5) through (10), and further nonnegativity constraints on the deviations.

Using the MOBLP procedure, the bank planning model can be solved in its original form. Since increases in profits are desirable, but increases in the CA and RA objectives are not, the model is specified to maximize Equation (2) and minimize Equations (3) and (4) subject to the constraints in (5) through (10).

Solutions and Discussion

The ultimate goal of decision-makers, when faced with multiple conflicting objectives, is to consider the tradeoffs between the objectives and to reach a decision that is most suitable to their subjective preferences. Both GP and MOBLP allow consideration of these tradeoffs, although in quite different manners. To present comparisons of the two methods, the numerical example is solved first as a GP, then as a MOBLP problem.

GP Solutions and Decision Process

To solve the GP problem, assume that bank managers determine that a profit goal of $4 million is appropriate. Further assume that management determines that desirable goals are 1.1 and 7.5 for the CA and RA objectives, respectively. Finally, management considers overachievement and underachievement equally important for all three objectives, i.e., the priority weights for the problem are equal. Given these goals and weights, the GP solution is shown in Exhibit 2 as solution 1. The GP model yields a profit of $4.38 million and CA and RA ratios of 1.1 and 7.5, respectively. Thus, the decision-maker finds a single "optimal" solution.

If managers are reasonably confident of their original goals and priorities, they may be satisfied with this GP solution, since it allows them to achieve liquidity and risk objectives while at the same time making greater profits than expected. If, however, bank managers find it difficult to formulate goals for the objectives, it may be necessary to solve the GP problem a number of times using different desired values for the objectives and/or different priority weights. To implement this procedure, a number of different goal values and weights were chosen and the GP problem re-solved. These solutions are shown in
Next, the problem was solved as a MOBLP problem using the algorithm developed by Milan Zeleny [15]. The Zeleny algorithm yielded 17 non-dominated extreme point solutions to the numerical example. Exhibit 3 presents the objective function values for 7 of those solutions. The other 10 solutions are not presented since the value for at least one of the objectives is greatly exaggerated beyond the realm of possible choice. Solutions of this type can be eliminated, of course, by additional constraints.

As Exhibit 3 shows, the MOBLP procedure yields a wide range of efficient solutions to the problem. The objective values range from a high of $5.19 million for profit and accompanying CA and RA ratios of 1.40 and 9.12, respectively, to a low of $4.39 million for profit and CA and RA ratios of .75 and 6.46, respectively. It should be evident that none of the solutions shown in Exhibit 3 is objectively the "best" solution, since an improvement in any objective requires sacrifices in other objectives. The results of the MOBLP model present the decision-maker with an efficient set of solutions from which to choose. What management chooses will depend on the subjective preferences of bank managers and on their evaluation of the explicit tradeoffs derived from comparisons of the solutions.

Comparison of the Numerical Results

The goal is to choose the combination of profit, liquidity, and risk that most suits the subjective preferences of bank managers. If GP is used to solve the problem, and managers are satisfied with the original goal values used to find solution 1 in Exhibit 2, this solution may be chosen as the final plan of action. There are other feasible courses of action, however, that are clearly superior. The MOBLP solutions 3, 4, and 6 are all more desirable in terms of the three objectives, i.e., they are more profitable, more liquid, and less risky. By limiting themselves to the inefficient GP solution, bank managers incur considerable costs. They expose themselves to greater risk of insolvency, and they earn a lower rate of return on equity.

The problem with the GP solution is that it requires decision-makers to choose desired values for the objectives and priorities to be attached to these objectives prior to any knowledge of the feasible set of solutions. Since the decision-maker does not know his feasible alternatives, he may choose values that will lead to suboptimal solutions. Moreover, there is no clear-cut way to determine whether the GP solution is suboptimal without knowledge of the efficient set. The problem is further compounded when larger, more sophisticated models are employed.

If alternative solutions are found with GP, as in solutions 2 through 5 in Exhibit 2, even these solutions may be imperfect. Each one is inferior to at least one of the MOBLP solutions presented in Exhibit 3. Since MOBLP locates solutions that are always efficient, GP cannot locate a superior solution and may definitely lead to inferior solutions.

Implications and Conclusions

Decision-makers faced with multiobjective problems may take two possible approaches to a solution: 1) direct utility maximization, which involves specifying certain aspects of the utility function, and 2) an indirect approach, which involves finding and examining the efficient set of solutions to the problem. GP is
most closely associated with the former approach since utility parameters must be included in the problem through goal values and priority weights. Since these goal values and weights are difficult, if not impossible, to accurately specify, the solutions found with the use of GP may not be efficient and thus not optimal. Even though GP can be used to examine the efficient set of solutions, as some writers have suggested, it is not well suited for such a procedure since the solutions may or may not be optimal. MOBLP, following the latter approach, is ideally suited for exploring the efficient set of solutions without requiring the explicit introduction of the utility function into the model. The solutions found by MOBLP are always efficient, ensuring the best choices.

The numerical problem presented in this paper demonstrates how a GP model of a simple bank balance sheet optimization problem can lead to inefficient solutions. MOBLP is a superior decision-making technique for multiobjective problems. It should prove to be a powerful tool for a number of financial problems.

References